A PARSIMONIOUS CHOQUET MODEL OF SUBJECTIVE LIFE EXPECTANCY

Alexander Ludwig and Alexander Zimper

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Center for Retirement Research at Boston College
Hovey House
140 Commonwealth Avenue
Chestnut Hill, MA 02467
Tel: 617-552-1762 Fax: 617-552-0191

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Center for Retirement Research at Boston College
Hovey House
140 Commonwealth Avenue
Chestnut Hill, MA 02467
phone: 617-552-1762 fax: 617-552-0191
e-mail: crr@bc.edu
www.bc.edu/crr

Affiliated Institutions:
The Brookings Institution
Massachusetts Institute of Technology
Syracuse University
Urban Institute

Abstract

This paper develops and estimates a closed-form model of Bayesian learning of subjective survival beliefs within the framework of Choquet decision theory. Data from the Health and Retirement Study (HRS) indicate that, on average, young respondents underestimate their true survival probability whereas old respondents overestimate their survival probability. Such subjective beliefs violate the rational expectations paradigm and are also not in line with the predictions of the *rational* Bayesian learning paradigm which implies convergence of subjective to underlying 'objective' probabilities. Based on the assumption of non-additive beliefs, we therefore introduce a model of Bayesian learning which combines rational learning with the possibility that the interpretation of new information is prone to psychological attitudes. We estimate the parameters of our theoretical model by pooling the HRS data. Despite a parsimonious parametrization we find that our Choquet model results in a remarkable fit to the average subjective beliefs expressed in the data.

1 Introduction

Dynamic economic models are based on the forward looking behavior of economic agents. In the context of life-cycle models, an individual's consumption and savings decision depends on her subjective beliefs about future interest rates, wage rates and the likelihood of dying. According to these models, individuals have beliefs about such variables and use these beliefs to make decisions today. Until recently, common practice in such studies was to assume rational expectations implying that the individuals' beliefs are given as objective probability distributions. The use of objective distributions is by now put into question by numerous researchers who suggest to directly measure subjective expectations and to evaluate the consequences of deviations of subjective expectations from their objective counterparts. Manski (2004) provides an overview on this literature.

Key for understanding life-cycle consumption and savings decisions, is an understanding of how individuals form survival expectations. The main contribution of the present paper is the introduction of a closed-form model of Bayesian learning of survival expectations within the framework of *Choquet decision theory* Schmeidler (1986, 1989) and Gilboa (1987). While Choquet decision theory has mainly been developed in order to model the decision behavior of individuals who commit Ellsberg paradoxes (Ellsberg 1961), our approach demonstrates the usefulness of Choquet decision theory in describing the learning behavior of a representative agent with respect to her subjective survival beliefs. As our point of departure, we present stylized facts on a comparison between average subjective survival expectations from the HRS and their objective counterparts that can be summarized as follows: First, on average, individuals of relatively young age underestimate survival probabilities whereby "young" refers to age bands from about 50 to 70 in our data. Second, this "pessimistic" bias monotonically decreases with age to zero for respondents of about age 70. Third, old respondents, that is, individuals of about age 70 and older, overestimate their actual survival probability. Fourth, this "optimistic" bias monotonically increases with age. Finally, the initial pessimistic bias is slightly higher for women and the final optimistic bias is slightly higher for men.[1]

As we argue in this paper, these stylized facts are incompatible with the rational expectations paradigm. Furthermore, the observed age-dependent biases in the data also suggest a violation of the rational Bayesian learning paradigm. Models of subjective belief formation based on rational Bayesian learning generate posterior beliefs that are closer to the true, i.e., objective, distribution the more experienced the agent becomes. If an agent gains more experience by getting older, rational Bayesian learning requires the agent to learn with increasing age the true probabilities, cf. Viscusi (1990, 1991). Under

[1]Our findings thus confirm similar results documented in a literature initiated by Hammermesh (1985). In two different data samples from surveys, Hammermesh (1985) found that people do incorporate improvements of life-expectancy into their beliefs about personal longevity and that the subjective survival curves are somewhat flatter than the objective data. Similar differences between subjective beliefs and the objective data have been reported for the HRS by Hurd and Kathleen (1995) and Gan, Hurd, and McFadden (2003) and others and, more recently, for the Survey of Health, Ageing and Retirement in Europe (SHARE) data (Hurd, Rohwedder, and Winter 2005).

the assumption of rational Bayesian learning any gap between subjective beliefs and objective survival probabilities should therefore decrease with increasing age, implying that the average beliefs of people are closer to the true probabilities when they get older.

To match the key stylized facts in the data – in particular, pessimism at young and optimism at old age – we develop a Bayesian learning model of survival beliefs that differs from the existing literature on subjective expectations in two important respects. First, we allow for the possibility that people report subjective beliefs that express ambiguity attitudes. We formally describe such ambiguous beliefs as non-additive probability measures, i.e., *capacities*, which arise in Choquet decision theory. More specifically, we use so-called *neo-additive capacities*, as introduced by Chateauneuf, Eichberger, and Grant (2007), according to which a Choquet decision maker resolves her ambiguity by focussing on the best, resp. worst, possible consequence of her action. Second, as a generalization of the standard assumption of rational Bayesian learning, we consider a model of psychologically biased Bayesian learning in which neo-additive beliefs are updated in accordance with the *generalized Bayesian update rule* (Eichberger, Grant, and Kelsey 2006). For the representative agent of our model an initial bias between her subjective beliefs and objective probabilities does not necessarily vanish in the long run. Several studies in the psychological literature show that real-life agents systematically violate the assumption of rational Bayesian learning in that their learning behavior is prone to effects such as "myside bias" or "irrational belief persistence" cf. Baron (2007, Ch. 9) and the references cited there. The stylized facts in our data on subjective survival beliefs may reflect such attitudes and our formal approach accommodates these.

Our theoretical framework provides a parsimonious specification of the representative agent's age-belief pattern with three parameters, reflecting, first, an initial bias in subjective survival probabilities, second, a measure for the agent's ambiguity with respect to her initial estimator of her subjective survival probability, and, third, the degree of optimism, respectively pessimism, by which the agent resolves her ambiguity. We then estimate the parameters of our Choquet model by pooling the HRS data. Despite the low parametrization, our model results in a decent fit to the average data on subjective beliefs. We also find that the model's performance is somewhat better for female than for male respondents.

Our approach is related to a literature initiated by Viscusi (1985) who analyzes changes in risk perceptions by a simple model of rational Bayesian learning. In the context of the HRS data on subjective survival probabilities this approach has been used by Smith, Taylor, and Sloan (2001) and Smith, Taylor, Sloan, Johnson, and Desvouges (2001) who test how new information about health shocks between two interview waves affects updating of individuals in the HRS.[2] In contrast to this literature our approach is more general in that it allows for the possibility that individuals are not rational Bayesian learners. Moreover, while we do not investigate how certain idiosyncratic shocks, e.g., general versus smoking

[2]Under the assumption that all individuals are rational Bayesian learners, Smith, Taylor, Sloan, Johnson, and Desvouges (2001) show that a representative smoker updates her beliefs in a different way than a representative non-smokers when she learns about "general" compared to "smoking-related" health shocks.

related health shocks or parental death, affect updating of individuals and how updating differs across a variety of subgroups in the population, the strength of our parsimonious setup is that it can be directly mapped into calibrated micro- or macroeconomic life-cycle models with representative agents.

The remainder of our analysis is structured as follows. Section 2 documents the stylized facts in the HRS data. Section 3 presents our decision theoretic framework resulting in a parsimonious model of subjective life expectancy. We then present the main results of our empirical analysis in section 4. Section 5 provides a discussion of our results. Finally, section 6 concludes. Furthermore, appendix A provides additional analytical results and appendix B contains a detailed description of our data.

2 Stylized Facts

We compare subjective survival beliefs, based on the data of the Health and Retirement Study (HRS), with objective survival rates. In our data on subjective beliefs we have information about individuals' expectations to live from the age at interview j up to some target age m. Age at interview j and target age m are assigned according to the pattern in table 1. Our objective survival rates are based on cohort life tables for the U.S. population. A detailed description of our data sources and methods is provided in appendix B. The following section only provides a brief summary.

Table 1: Interview and Target Age

Age at Interview j	Target Age m
≤69	80
70-74	85
75-79	90
80-84	95
85-89	100

Source: RAND HRS Data Documentation, Version F (October 2006).

2.1 The Data

In the HRS, respondents of waves 5 through 7 were asked in the respective interview years 2000, 2002 and 2004 about their probability to live from interview age j until a certain target age m, cf. table 1. In our analysis, we pool the information in these three waves. As we discuss in appendix B.1, we do not consider households of age $40-49$ and of age 90 and older. In addition we exclude some observations with inconsistent answering patterns. This selection by age and consistency of answering patterns leaves us with a total sample size of 44671 observations out of which 18341 are male and 26330 are female respondents. We refer to this sample as our "full sample". While most of our analysis focusses on this full sample, we further investigate the sensitivity of our results with respect to focal point answers at subjective survival probabilities of 0, 50, and 100 percent in subsection 4.3.

As we compare the subjective survival probabilities to their objective counterparts, we next have to construct objective survival rates. In correspondence with our representative agent model that we develop in section 3, we follow the literature initiated by Hammermesh (1985) and use cohort life tables for the entire U.S. population as the objective data. To construct those we predict future survival rates in the population. Our estimates are based on data for age-specific survival rates for the years 1900 to 2004 taken from the Human Mortality Database (2008) (HMD) and the Social Security Administration (SSA). Since projections from official sources tend to underestimate future increases in survival probabilities, we do not use SSA cohort life tables but rather base the prediction of future survival rates on a Lee-Carter procedure (Lee and Carter 1992). The idea of our approach is that agents in our model base their predictions of their respective objective survival probabilities on past data but it is unobserved to the econometrician which point estimates they use. For this reason we account for the uncertainty of the objective data in the estimation of standard errors, cf. section 4. As an additional advantage, our procedure assigns the objective information on survival rates in correspondence with the HRS interview years.

2.2 Illustration

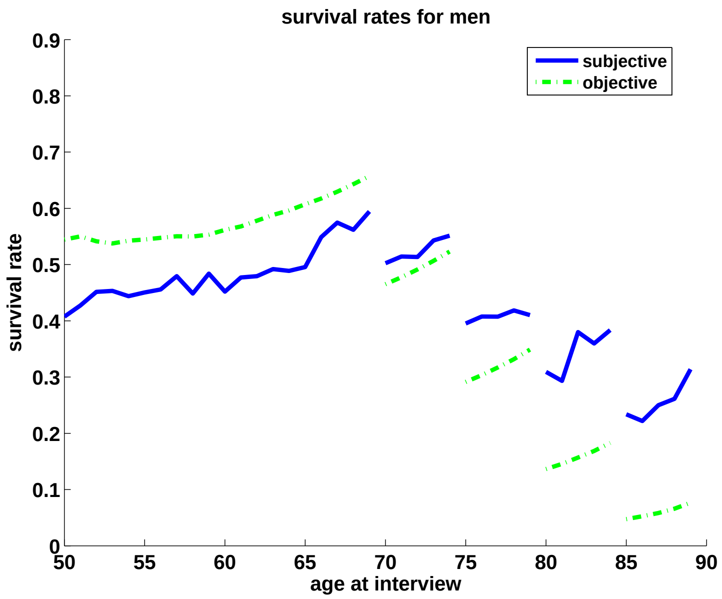
Figure 1 summarizes the information in our data by displaying the average subjective beliefs on survival of HRS respondents against the age at interview and the respective objective data for men in panel (a) and women in panel (b). The different line segments are due to changes in target ages, cf. table 1. Two stylized facts emerge for either gender from the data. First, the subjective beliefs on survival are downward biased at younger ages. Second, the subjective beliefs on survival are upward biased at older ages whereby the upward bias increases with age. These stylized facts clearly indicate a systematic violation of the rational expectations paradigm of economic theory by which there should be no difference between subjective beliefs and objective survival rates.

For younger respondents (≤ 69) the data in figure 1 is compatible with the convergence behavior as predicted by rational Bayesian learning.[3] However, upon inspection of the age-belief pattern of elderly respondents of age 75 and older in figure 1, the picture changes. In figure 2 we zoom in from figure 1 the average beliefs of male respondents between interview ages 80 to 89 to survive until 95, respectively until 100, against their objective counterparts in panel (a). To illustrate learning behavior in this age group we estimated simple linear trends for both the subjective and the objective data and display the differences in these trends in panel (b) of the same figure. These graphs

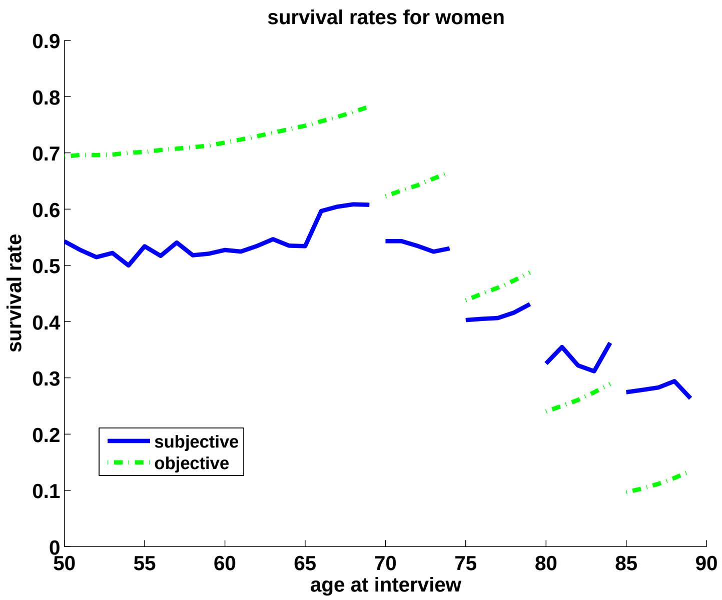
[3]For women we do not observe such a clear convergent pattern even for this age group.

Figure 1: Subjective and objective survival probabilities

(a) Men



(b) Women



Source: Own calculations based on HRS, HMD and SSA data.

indicate divergence with increasing age. This divergent pattern is stronger for the higher interview/target age group. Thus, contrary to the predictions of the rational Bayesian learning model the average bias between subjective beliefs and objective probabilities increases rather than decreases with more experience whereby this effect appears to be stronger for higher target ages.

The patterns shown in figure 2 illustrate a violation of the rational Bayesian learning paradigm within target age groups. Furthermore, notice that, in order to explain the data across target age groups, the rational Bayesian learning hypothesis would require highly implausible prior beliefs. For example, the overestimation of the subjective belief of an 80 year old agent to live until 95 by 17.28 percentage points for men (8.54 percentage points for women), cf. figure 1, can only be explained with rational Baysian learning if the same agent expressed a prior belief with a much higher degree of overestimation about her survival at the age of 50. However, at the age of 50, we actually observe an average underestimation of the survival belief by -13.70 percentage points for men (-15.07 percentage points for women).

Our model of Bayesian learning with psychological bias captures the stylized facts of figure 1 in a very parsimonious way whereby it also offers a plausible explanation why young people are too pessimistic whereas elderly people are too optimistic about their survival expectations. While rational Bayesian learning may be appropriate in situations in which individuals are emotionally detached from the arrival of new information (think, e.g., about tossing a coin in order to learning the odds whether it ends up heads or tails), this may not be the case if new information has a strong personal impact on the individual. In such situations the individual's learning process may be prone to emotions such as hope or despair. This holds in particular true when an individual learns new information about its life expectancy thereby facing the prospect of its own death.[4] We feel that the most plausible explanation for the overly optimistic life expectations of elderly people is an optimistic "myside bias" in their interpretation of new information which assists them to better ignore the increasingly relevant prospect of death. In contrast, younger people are less biased because the prospect of their death is less relevant yet and they may even underestimate increases in life expectancy due to medical progress.

3 A Parsimonious Model of Subjective Life Expectancy

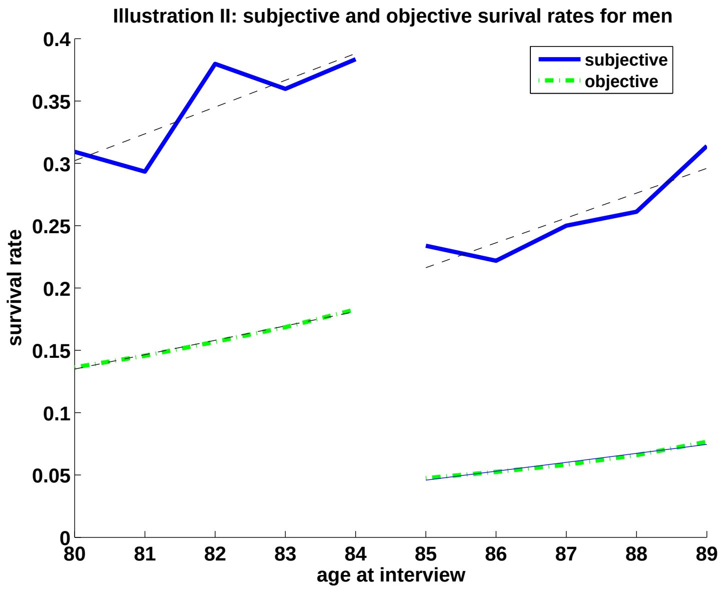
3.1 Ambiguous Beliefs

We assume that individuals exhibit *ambiguity attitudes* in the sense of Schmeidler (1989) and who may thus, for example, commit paradoxes of the Ellsberg type (Ellsberg 1961). Following Schmeidler (1989), such individuals could be described as Choquet Expected Utility (CEU) decision makers, that is, they maximize expected utility with respect to

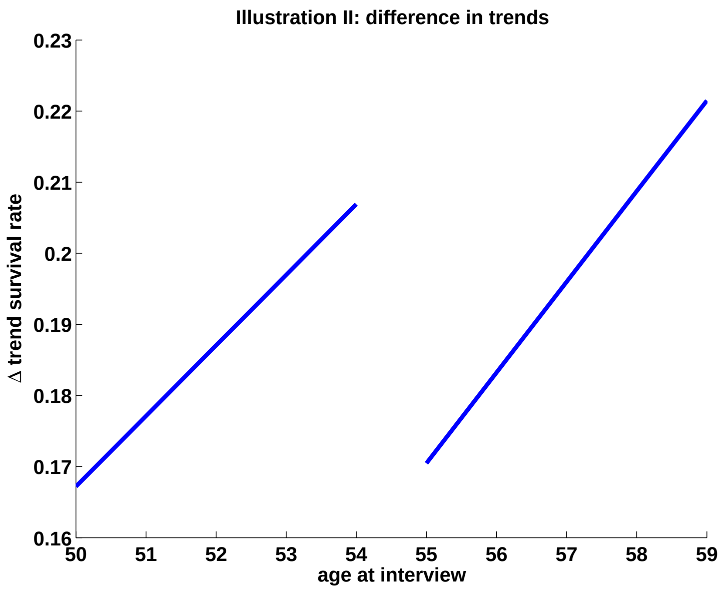
[4]Along this line, Kastenbaum (2000) summarizes the insights of psychological research on the reflection about personal death as follows: "There are divergent theories and somewhat discordant findings, but general agreement that most of us prefer to minimize even our cognitive encounters with death."

Figure 2: Survival probabilities at age 80 and older for men

(a) Objective and subjective probabilities



(b) Difference in trends



Notes: The dashed lines in panel (a) are predicted values from a simple linear trend estimation. Panel (b) displays the difference in these linear trends.
Source: Own calculations based on HRS, HMD and SSA data.

non-additive beliefs.[5] Our own approach focuses on non-additive beliefs that are defined as *neo-additive capacities* in the sense of Chateauneuf, Eichberger, and Grant (2007).

Definition 1. *For a given measurable space* $(\Omega, \mathcal{E})$ *the neo-additive capacity,* ν*, is defined, for some* $\delta, \lambda \in [0,1]$ *by*

$$\nu(A) = \delta \cdot (\lambda \cdot \omega^o(A) + (1-\lambda) \cdot \omega^p(A)) + (1-\delta) \cdot \mu(A) \tag{1}$$

for all $A \in \mathcal{E}$ *whereby* $\mu(\pi)$ *is some additive probability measure and we have for the non-additive capacities* ω^o

$$\begin{aligned} \omega^o(A) &= 1 \text{ if } A \neq \emptyset \\ \omega^o(A) &= 0 \text{ if } A = \emptyset \end{aligned}$$

and ω^p *respectively*

$$\begin{aligned} \omega^p(A) &= 0 \text{ if } A \neq \Omega \\ \omega^p(A) &= 1 \text{ if } A = \Omega. \end{aligned}$$

For a real-valued Savage act f with closed and bounded range, it can be shown that the Choquet expected utility of f with respect to a neo-additive capacity ν is given as

$$\begin{aligned} E(u(f), d\nu) &= \delta \cdot (\lambda \cdot \max u(f(\cdot)) + (1-\lambda) \cdot \min u(f(\cdot))) \\ &\quad + (1-\delta) \cdot E(u(f), d\mu), \end{aligned} \tag{2}$$

where $u : X \to \mathbb{R}$ is a von Neumann-Morgenstern utility function and $E(u(\cdot), d\nu)$ is the Choquet expected value of $u(\cdot)$ with respect to ν, cf. Schmeidler (1986). Neo-additive capacities can be interpreted as non-additive beliefs that represent deviations from additive beliefs whereby a parameter δ (*degree of ambiguity*) measures the lack of confidence the decision maker has in some subjective additive probability distribution μ. Obviously, if there is no ambiguity, i.e., $\delta = 0$, equation (2) reduces to the standard subjective expected utility representation $E(u(f), d\mu)$ of Savage (1954). In case there is some ambiguity, however, the second parameter λ measures how much weight the decision maker puts on the best possible outcome of alternative f when resolving her ambiguity. Conversely, $(1-\lambda)$ is the weight she puts on the worst possible outcome of f. As a consequence, we interpret λ as an "optimism under ambiguity" parameter.

In the context of survival expectations, we are interested in the agent's belief to be alive at some target age m. Let us misuse notation and also write m for the event that

[5] Properties of non-additive beliefs are used in the literature for formal definitions of, e.g., *ambiguity* and *uncertainty attitudes* (Schmeidler 1989; Epstein 1999; Ghirardato and Marinacci 2002), *pessimism* and *optimism* (Eichberger and Kelsey 1999; Wakker 2001; Chateauneuf, Eichberger, and Grant 2007), as well as *sensitivity to changes in likelihood* (Wakker 2004).

the agent is still alive at age m. Under the assumption that there is always the possibility to reach age m, the event m cannot be the null event, implying $\omega^o(m) = 1$. On the other hand, we also stipulate that there is always the possibility to die before reaching age m so that m cannot be the universal event either, implying $\omega^p(m) = 0$. As a consequence, the agent's belief to survive until age m in (1) simplifies to

$$\nu(m) = \delta \cdot \lambda + (1 - \delta) \cdot \mu(m). \tag{3}$$

According to our interpretation, the additive probability $\mu(\cdot)$ in (3) stands in for the agent's "rational" part of her survival beliefs. Under the *rational expectations* paradigm the subjective additive probability measure μ must, first, coincide with the "true" probability distribution and, second, the agent must not be ambiguous about her subjective belief, i.e., $\delta = 0$. However, we do not only assume that the representative agent is ambiguous about her subjective belief, $\delta \neq 0$, but also that the subjective probability μ may deviate from its objective counterpart.

3.2 The Benchmark Case: Rational Bayesian Learning

Consider the situation of an agent who is uncertain about the probability that individuals of age j survive until age m. We assume that the agent receives information which is equivalent to a statistical experiment in which n individuals of age j are independently drawn so that the agent observes for every individual whether it survives until m or not.

Formally, we consider the state space

$$\Omega = [0, 1] \times S^\infty$$

with $S^\infty = \times_{i=0}^{\infty} \{m, \bar{m}\}$ where m and $\bar{m}$ are possible outcomes in each trial ($\bar{m}$ is death before age m). As event space we define $\mathcal{E} = \mathcal{B} \times \mathcal{S}^\infty$ whereby $\mathcal{B}$ denotes the Borel σ-algebra of the unit-interval $[0, 1]$ and $\mathcal{S}^\infty$ denotes the powerset of S^∞, i.e., $\mathcal{S}^\infty = 2^{S^\infty}$. Let $\pi \in \mathcal{E}$ denote the event that $\pi_{j,m} \in [0, 1]$ is the probability of outcome m. In our framework $\pi_{j,m}$ is the parameter of a Binomial-distribution that stands in for the objective probability of survival from j to m, i.e., the probability that outcome m occurs. For notational convenience, we will drop in this and the subsequent subsection subscripts and simply write π instead of $\pi_{j,m}$. Similarly, let $\mathbf{I}_n^k \in \mathcal{E}$ denote the event that outcome m has occurred k-times in the first n trials. More specifically, we assume that $\mathbf{I}_n^k \in \mathcal{F}_n$ for $n \geq 0$, whereby $\mathcal{F}_n$ is defined as the σ-algebra generated by the collection of so-called *n-rectangle sets*

$$[0, 1] \times A_1 \times ... \times A_n \times S^\infty \in \mathcal{E}$$

where $A_i \in 2^{\{m,\bar{m}\}}$ for $i = 0, ..., n$. Obviously, $\mathcal{F}_0 \subseteq \mathcal{F}_1 \subseteq ... \subseteq \mathcal{F}_\infty$ so that the $(\mathcal{F}_n)_{n \geq 0}$ constitute a *filtration*, which – together with the fact that $\mathbf{I}_n^k \in \mathcal{F}_n$ but $\mathbf{I}_n^k \notin \mathcal{F}_{n-1}$ – formally captures the idea that the agent receives more information with increasing sample size n.

Let us now suppose that the agent has a subjective additive probability measure, $\mu_{j,m}$, defined on the probability space $(\Omega, \mathcal{E})$ whereby, for instance, $\mu_{j,m}\left(\pi \cap \mathbf{I}_n^k\right)$ denotes the agent's subjective (joint) probability that the "true" probability of survival from j to m is π and that she will observe information $\mathbf{I}_n^k$. Again, we drop the subscripts and write μ instead of $\mu_{j,m}$. While the true value, say π^*, of parameter π is not known to the agent, we further assume that the agent's prior over the random variable π is given as a Beta distribution so that her estimator for the true probability of m, i.e., π^*, is the expected value of this Beta-distribution, i.e., $E[\pi, d\mu] = \frac{\alpha}{\alpha+\beta}$ for given distribution parameters $\alpha, \beta > 0$. That is, the prior distribution over π is characterized by the probability density[6]

$$\mu(\pi) = \begin{cases} K_{\alpha,\beta}\pi^{\alpha-1}(1-\pi)^{\beta-1} & \text{for } 0 \leq \pi \leq 1 \\ 0 & \text{else} \end{cases}$$

where $K_{\alpha,\beta}$ is a normalizing constant.[7] Since the probability of receiving information $\mathbf{I}_n^k$ for a given π (=likelihood function) is, by the Binomial-assumption,

$$\mu\left(\mathbf{I}_n^k \mid \pi\right) = \binom{n}{k}\pi^k(1-\pi)^{n-k},$$

we obtain by Bayes' rule

$$\begin{aligned} \mu\left(\pi \mid \mathbf{I}_n^k\right) &= \frac{\mu\left(\pi \cap \mathbf{I}_n^k\right)}{\mu\left(\mathbf{I}_n^k\right)} \\ &= \frac{\mu\left(\mathbf{I}_n^k \mid \pi\right)\mu(\pi)}{\int_{[0,1]}\mu\left(\mathbf{I}_n^k \mid \pi\right)\mu(\pi)\,d\pi} \\ &= K_{\alpha+k,\beta+n-k}\pi^{\alpha+k-1}(1-\pi)^{\beta+n-k-1}. \end{aligned}$$

Observe that the agent's subjective posterior distribution over π is a Beta-distribution with parameters $\alpha+k, \beta+n-k$. Accordingly, the agent's posterior belief is given by the expected value of the posterior distribution, $E\left[\pi, d\mu\left(\cdot \mid \mathbf{I}_n^k\right)\right] = \frac{\alpha+k}{\alpha+\beta+n}$, which, using that the prior belief is $E[\pi, d\mu] = \frac{\alpha}{\alpha+\beta}$, we can rewrite as

$$E\left[\pi, d\mu\left(\cdot \mid \mathbf{I}_n^k\right)\right] = \left(\frac{\alpha+\beta}{\alpha+\beta+n}\right)E[\pi, d\mu] + \left(\frac{n}{\alpha+\beta+n}\right)\frac{k}{n} \tag{4}$$

where $\frac{k}{n}$ is the sample mean. That is, the agent's posterior belief about the probability of m is a weighted average of her prior and the sample mean whereby the weight attached to the sample mean increases in the number of trials.[8] Since, for every $c > 0$,

[6]For the ease of exposition we somewhat abuse notation in that we write μ interchangeably for an additive probability measure, which is a set function, and for a density function, which is defined on the real line.

[7]In particular, $K_{\alpha,\beta} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$ where $\Gamma(y) = \int_0^\infty x^{y-1}e^{-x}dx$ for $y > 0$.

[8]Tonks (1983) introduces a similar model of rational Bayesian learning in which the agent has a normally distributed prior over the mean of some normal distribution and receives normally distributed information.

$\lim_{n\to\infty} prob\left(\left|\frac{k}{n}-\pi^*\right|\leq c\right)=1$ we obtain the following result for this standard model of rational Bayesian learning.

Observation 1. *Under rational Bayesian learning the agent's estimator* $E\left[\pi, d\mu\left(\cdot \mid \mathbf{I}_n^k\right)\right]$ *converges in probability to the true probability* π^* *that individuals of age* j *will survive until age* m *if the number of trials,* n*, approaches infinity.*

3.3 Bayesian Learning with Psychological Bias

In this subsection we develop our concept of Bayesian learning with a psychological bias as a generalization of the rational learning model discussed above. In our model the representative agent receives new information about her life-expectancy when she gets older.

Consider now the case in which the agent is a Choquet decision maker so that her prior beliefs about π are given by a neo-additive capacity (3) whose additive part is described by a Beta-distribution μ. Her (prior) estimator is now the Choquet expected value of $\pi\in[0,1]$ with respect to the non-additive prior ν

$$\begin{aligned} E[\pi, d\nu] &= \delta\cdot(\lambda\cdot\max\pi+(1-\lambda)\cdot\min\pi)+(1-\delta)\cdot\frac{\alpha}{\alpha+\beta} \\ &= \delta\cdot\lambda+(1-\delta)\cdot E[\pi, d\mu]. \end{aligned}$$

Under the assumption that the agent updates (3) in the light of new information $\mathbf{I}_n^k$, her posterior beliefs are given by the conditional non-additive probability measure $\nu\left(\cdot \mid \mathbf{I}_n^k\right)$ so that the (posterior) estimator of π becomes $E\left[\pi, d\nu\left(\cdot \mid \mathbf{I}_n^k\right)\right]$.

At this point we have to take a stand on how an agent updates her ambiguous beliefs. Several different Bayesian update rules are perceivable for the non-additive beliefs of CEU decision-makers (Gilboa and Schmeidler 1993; Sarin and Wakker 1998; Pires 2002; Eichberger, Grant, and Kelsey 2006; Siniscalchi 2001; Siniscalchi 2006). In this paper we consider the so-called generalized Bayesian update rule for which we derive in appendix A.1 the neo-additive posterior belief $\nu\left(\cdot \mid \mathbf{I}_n^k\right)$. Applied to survival beliefs we then obtain the following neo-additive (posterior) estimator that the agent of age j will be alive at age m given her information $\mathbf{I}_n^k$

$$E\left[\pi, d\nu\left(\cdot \mid \mathbf{I}_n^k\right)\right]=\delta_{\mathbf{I}_n^k}\cdot\lambda+\left(1-\delta_{\mathbf{I}_n^k}\right)\cdot E\left[\pi, d\mu\left(\cdot \mid \mathbf{I}_n^k\right)\right]$$

where

$$\delta_{\mathbf{I}_n^k}=\frac{\delta}{\delta+(1-\delta)\cdot\mu\left(\mathbf{I}_n^k\right)}.$$

Furthermore, we show in appendix A.2 that the unconditional probability of receiving information $\mathbf{I}_n^k$ is given by

$$\begin{aligned} \mu\left(\mathbf{I}_n^k\right) &= \frac{\mu\left(\mathbf{I}_n^k \mid \pi\right)\mu(\pi)}{\mu\left(\pi \mid \mathbf{I}_n^k\right)} \\ &= \binom{n}{k}\frac{(\alpha+k-1)\cdot\ldots\cdot\alpha\cdot(\beta+n-k-1)\cdot\ldots\cdot\beta}{(\alpha+\beta+n-1)\cdot\ldots\cdot(\alpha+\beta)}. \end{aligned}$$

In a final step, we link the information received by the agent to her age. We suppose that an agent of age h receives information $\mathbf{I}^k_{n(h)}$ which is equivalent to information gained from a statistical experiment with $n(h)$ trials whereby the *experience* function $n(h)$ satisfies $n(0) = 0$, $n(h) < n(h+1)$ for all h and $n(h) \to \infty$ if $h \to \infty$. That is, our approach associates a higher age with greater experience whereby we do not restrict the gaining of experience by any upper bound. The following proposition summarizes the considerations from above.

Proposition 1. *Under the assumption of Bayesian learning with psychological bias, the posterior belief of an agent of age h to survive from age j to age m conditional on the information $\mathbf{I}^k_{n(h)}$ is given by*

$$E\left[\pi, d\nu\left(\cdot \mid \mathbf{I}^k_{n(h)}\right)\right] = \delta_{\mathbf{I}^k_{n(h)}} \cdot \lambda + \left(1 - \delta_{\mathbf{I}^k_{n(h)}}\right) \cdot E\left[\pi, d\mu\left(\cdot \mid \mathbf{I}^k_{n(h)}\right)\right]$$

whereby

$$\delta_{\mathbf{I}^k_{n(h)}} = \frac{\delta}{\delta + (1-\delta) \cdot \mu\left(\mathbf{I}^k_{n(h)}\right)}$$

with

$$\mu\left(\mathbf{I}^k_{n(h)}\right) = \binom{n(h)}{k} \frac{(\alpha + k - 1) \cdot \ldots \cdot \alpha \cdot (\beta + n(h) - k - 1) \cdot \ldots \cdot \beta}{(\alpha + \beta + n(h) - 1) \cdot \ldots \cdot (\alpha + \beta)} \tag{5}$$

and

$$E\left[\pi, d\mu\left(\cdot \mid \mathbf{I}^k_{n(h)}\right)\right] = \left(\frac{\alpha + \beta}{\alpha + \beta + n(h)}\right) E[\pi, d\mu] + \left(\frac{n(h)}{\alpha + \beta + n(h)}\right) \frac{k}{n(h)}$$

where $E[\pi, d\mu]$ is the agent's prior additive estimator of the conditional survival probability and $\frac{k}{n(h)}$ stands for the observed sample mean of individuals who have survived from age j to age m.

3.4 A Parsimonious Model

We now develop a highly simplified version of our model of Bayesian learning with psychological bias that we bring to the data on survival beliefs in section 4. To this end, we make the following assumptions:

Assumption 1. *The representative agent has a uniform prior distribution over the parameter π. That is, $\alpha = \beta = 1$, implying for (5)*

$$\begin{aligned} \mu\left(\mathbf{I}^k_{n(h)}\right) &= \binom{n(h)}{k} \frac{k!\,(n(h) - k)!}{(n(h) + 1) \cdot n(h)!} \\ &= \frac{1}{n(h) + 1}. \end{aligned} \tag{6}$$

Under assumption 1 the impact of the received information on the ambiguity part is independent of the observed k and depends only on the number of observations $n(h)$.

Assumption 2. *We suppose that the representative agent observes at every age sample means that actually coincide with the objective survival rates, i.e., for all* h, $\frac{k}{n(h)}$ *coincides with the true survival probability* $\pi^*_{j,m}$ *to live from age* j *to* m*, whereby we re-introduce the subscript notation that had been dropped in the previous two subsections.*

Assumption 2 is, by the law of large numbers, appealing for a large number of observations, cf. observation 1.

Assumption 3. *We restrict ourselves to an experience function* $n(h) = h$ *whereby we assume that agents start learning at the age of* 20 *which corresponds to* $h = 1$ *in our model.*

The normalization of the initial age in assumption 3 corresponds with many life-cycle models of consumption and savings where agents are assumed to become economically active at the age of 20. In section 4.3 we investigate the sensitivity of our estimation results with respect to the initial age. There, we also address the sensitivity of our results with respect to the restriction of the experience function imposed by assumption 3.[9]

Assumption 4. *We initialize* $E[\pi_{r,r+1}, \mu]$ *for all ages* $r = j, \ldots, m-1$ *as*

$$E[\pi_{r,r+1}, d\mu] = \phi\pi^*_{r,r+1}. \tag{7}$$

Assumption 4 implies that the belief $E[\pi_{j,m}, d\mu]$ for all pairs (j, m) is given by $E[\pi_{j,m}, d\mu] = \prod_{r=j}^{m-1} \phi\pi^*_{r,r+1} = \phi^{m-j}\pi^*_{j,m}$.

Using assumptions 1 through 4 in proposition 1 we can summarize our parameterized Choquet model of subjective life expectancy as follows:

Proposition 2. *Let* $h \leq j < m$*. The posterior belief of an agent of age* h *to survive from age* j *to age* m *is*

$$E[\pi_{j,m}, d\nu(\cdot \mid h)] = \delta_h \cdot \lambda + (1 - \delta_h) \cdot E[\pi_{j,m}, d\mu(\cdot \mid h)] \tag{8}$$

whereby

$$\delta_h = \frac{\delta}{\delta + (1-\delta)\frac{1}{1+h}} \tag{9}$$

and

$$\begin{aligned} E[\pi_{j,m}, d\mu(\cdot \mid h)] &= \left(\frac{2}{2+h}\right) E[\pi_{j,m}, d\mu] + \left(\frac{h}{2+h}\right)\pi^*_{j,m} \\ &= \left(\frac{2\phi^{m-j} + h}{2+h}\right)\pi^*_{j,m}. \end{aligned} \tag{10}$$

[9] We experimented with a more general specification of the experience function as $n(h) = \psi h$, for some $\psi \geq 1$. However, parameters of our model were then only weakly identified.

Consequently, our simplified version of Bayesian learning with psychological bias results in a parsimonious specification of the representative agent's age-belief pattern with a vector of three parameters, $\Psi = [\phi, \delta, \lambda]$, only. These parameters reflect (i) an initial bias in the additive estimator reflecting overestimation, i.e., $\phi > 1$, or underestimation, i.e., $\phi < 1$, (ii) a measure for ambiguity, δ, and (iii) the degree of optimism, respectively pessimism, by which the agent resolves her ambiguity, λ.

If there is no ambiguity in the agent's beliefs, i.e., $\delta = 0$, our model reduces to a version of rational Bayesian learning by which the agent's subjective belief $E\left[\pi_{j,m}, d\nu\left(\cdot \mid h\right)\right]$ converges to the objective probability $\pi^*_{j,m}$ when her actual age h – and thereby the amount of gathered information – increases. Depending on an initial overestimation ($\phi > 1$), resp. underestimation ($\phi < 1$), the subjective beliefs thereby monotonically converge from "above", resp. "below", whereby this convergence behavior is the same for all target ages. As already discussed in the introduction, such a model of rational Bayesian learning can obviously not accommodate the stylized facts of figure 1, showing strong underestimation for a lower target age, e.g., $m = 80$, and strong overestimation for a higher target age, e.g., $m = 95$. In order to accommodate these stylized facts by rational Bayesian learning alone, an according model would require target-age specific parameters ϕ_m such that, e.g., $\phi_{85} < 1$ and $\phi_{95} > 1$. Such an extension would come at the cost of loosing parsimony without offering a straightforward interpretation of the additional parameters. In our opinion, it is therefore highly implausible that the HRS data may reflect rational Bayesian learning alone.

If, in contrast, there is some ambiguity involved, i.e., $\delta > 0$, our model implies that the impact of the additive part on the overall belief will decrease with increasing age under our assumption that the agents receive more information with increasing age. Observe that, for a given δ, δ_h is strictly decreasing in h whereby $\lim_{h\to\infty} \delta_h = 1$. That is, the older the agent gets, i.e., the more information she receives, the more is her survival belief determined by her optimistic/pessimistic attitude towards the resolution of ambiguity as expressed by parameter λ. In the context of survival expectations this convergence feature of our learning model allows us to formally express the idea that individuals minimize their "cognitive encounters with death" (Kastenbaum 2000) and suppress the notion of death the more relevant the risk of dying becomes, i.e., the older they are. The introduction of ambiguity thus results, in our closed-form learning model, in limit beliefs that converge, in general, not to the "objective" relative frequency π^* but rather enforce the agents' attitudes towards optimism/pessimism.

4 Empirical Analysis

4.1 Estimation Strategy

By proposition 2 we have to estimate three parameters, $\Psi = [\phi, \delta, \lambda]$. To estimate these parameters we pool a sample of the HRS data formed of the HRS waves $\{2000, 2002, 2004\}$.

Except for heterogeneity in sex and age, we ignore all other heterogeneity across individuals. We deliberately choose this strategy in order to focus the analysis on the main message of this paper: Choquet Bayesian learning is a more appropriate model for survival belief formation than rational Bayesian learning.[10] For notational convenience, we again do not display an index for sex.

In each interview age group j we have N_j observations denoted as $i \in \{1, \ldots, N_j\}$ where N_j differs across groups. In our estimation we weigh observations by the inverse of the group sizes, $\frac{1}{N_j}$, so that we down-weigh age groups with many observations relative to age groups with few observations and vice versa.[11] We assume a linearly additive error term and determine the parameter values by solving the following non-linear minimization problem

$$\min_{\Psi} \frac{1}{2} \sum_{j=1}^{J} \frac{1}{N_j} \sum_{i=1}^{N_j} \Big(E\left[\pi_{j,m}, d\nu_i\left(\cdot \mid h\right)\right] - \hat{E}\left[\pi_{j,m}, d\nu\left(\cdot \mid h\right)\right] \Big)^2 . \tag{11}$$

Here, $E\left[\pi_{j,m}, d\nu_i\left(\cdot \mid h\right)\right]$ denotes individual i's conditional subjective belief to survive from interview age j to target age m in the HRS data. $\hat{E}\left[\pi_{j,m}, d\nu\left(\cdot \mid h\right)\right]$ is the predicted subjective belief according to our model as described in proposition 2. Recall that target ages are assigned to interview ages according to the pattern in table 1.

We solve the above non-linear programming problem using a non-linear optimizer. As unique convergence is not guaranteed for such problems, we tried various combinations of starting values, Ψ_0, and alternative optimization routines for all of our scenarios that follow. For all these combinations the numerical routines returned the same solution vector $\hat{\Psi}$. We are therefore confident that the solvers converge to the unique global minimum. We bootstrap standard errors by drawing with replacement from our data on subjective beliefs and from our predicted data on objective survival probabilities, cf. appendix B.2, in 500 bootstrap iterations.

4.2 Main Results

Our main estimation results are summarized in table 2. For each estimated parameter, the table contains sex specific information on the point estimates, $\hat{\Psi}$, the respective standard errors, $\hat{\sigma}(\Psi)$, and the 95% confidence intervals of the coefficient estimates, $\widehat{CI}(\psi)$. In order to document the fraction of the overall variation of survival probabilities in the data that is explained by the respective parsimonious model we also report the R^2 of the

[10] In particular, it is not the purpose of this paper to analyze how updating of beliefs differs across particular idiosyncratic health shocks or other events that are regarded as relevant for survival belief formation in the literature, such as parental death. As we further discuss in our concluding remarks in Section 5, a more in depth analysis of survival belief formation based on idiosyncratic events using our framework is left for future research.

[11] Observe that this weighting scheme implies that our point estimates are identical to a regression based on the average survival rates in each group. Parameter estimates from an un-weighted regression are similar and are available upon request.

regressions. In addition, we report an "average R^2", denoted as $\bar{R}^2$, as a measure of the fraction of the variation in average survival probabilities explained by our model.

Table 2: Parameter estimates

	Men			Women		
	$\hat{\Psi}$	$\hat{\sigma}(\Psi)$	$\widehat{CI}(\psi)$	$\hat{\Psi}$	$\hat{\sigma}(\Psi)$	$\widehat{CI}(\psi)$
Initial bias: ϕ	0.891	0.002	[0.887 0.895]	0.900	0.002	[0.896 0.905]
Degree of ambiguity: δ	0.020	0.002	[0.017 0.024]	0.021	0.001	[0.019 0.023]
Degree of optimism: λ	0.454	0.012	[0.431 0.476]	0.394	0.012	[0.371 0.419]
R^2	0.041	0.003	[0.034 0.048]	0.063	0.003	[0.057 0.069]
$\bar{R}^2$	0.803	0.035	[0.691 0.834]	0.943	0.010	[0.905 0.944]

Notes: $\hat{\Psi}$ are point estimates of model parameters, $\hat{\sigma}(\Psi)$ is the respective standard deviation and $\widehat{CI}(\Psi)$ is the respective 95% confidence interval. Standard errors are calculated by bootstrapping the subjective and objective survival probabilities by drawing with replacement in 500 bootstrap iterations.
Source: Own calculations based on HRS, SSA and HMD data.

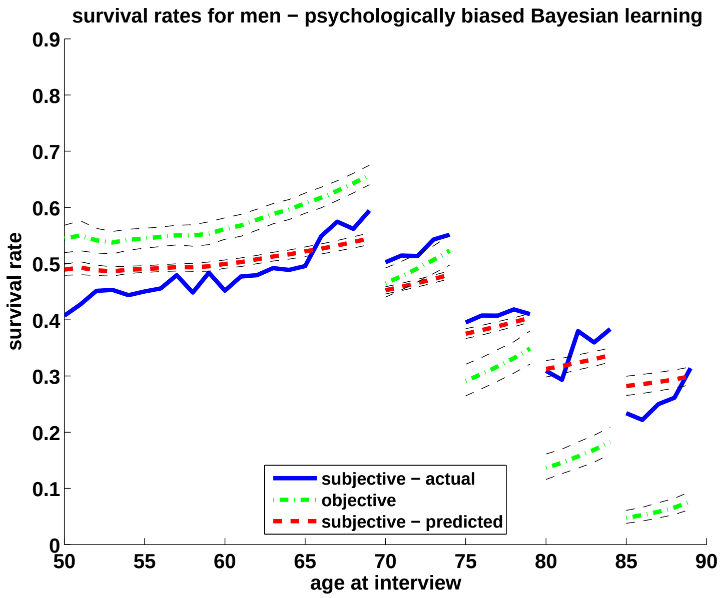
We have already argued that a model of rational Bayesian learning alone cannot explain the observed patterns in the data because predicted subjective survival rates from such a model would converge to the objective data, cf. observation 1. Quite in contrast, our model of psychologically biased Bayesian learning which considers ambiguous beliefs results in a decent fit to the average subjective survival expectations, also see figure 3. For both men and women, predicted subjective beliefs track the average subjective beliefs from the data nicely. The model explains 80% – 94% of the variation of average subjective survival rates whereby the fit is significantly higher in the case of women.[12] Unsurprisingly, our parsimonious specification of average beliefs results in low R^2s of the regressions – 4.1% for men and 6.3% for women – because our representative agent model can only capture some of the variation in answering patterns across individuals.

As far as the point estimates and the respective standard errors are concerned we first observe that all parameters are estimated with high precision. Accordingly, parameters δ and λ, which reflect the psychological biases in our model, are key for generating our results and we thereby formally reject the hypothesis of pure rational Bayesian learning. The point estimate of the initial bias, ϕ, is below one and captures the initial pessimism of subjective beliefs documented in figure 1. Interpretation of the point estimate of 0.89 for men (0.9 for women) is that a person without any experience estimates that the additive probability to survive from age 50 to age 80 – for which $m - j = 30$ – is only $\phi^{m-j} \cdot 100\% = 3.1\%$ (4.2%) of the actual objective probability. At the age of 20, with one

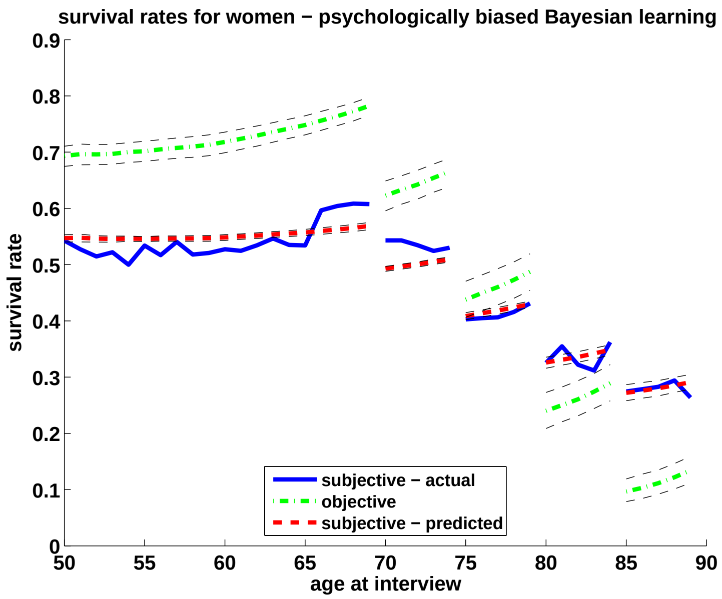
[12]The value of the two-sided t-test on the difference between the R^2s for men and women is 108.96 with a p-value of 0.0. The values of Jarque-Bera test statistics for normality of the distribution of the bootstrapped R^2s (and their p-values) are at 0.02 (0.99) for men and at 4.49 (0.10) for women so that a standard t-test is applicable.

Figure 3: Actual and predicted survival probabilities for psychologically biased Bayesian learning

(a) Men

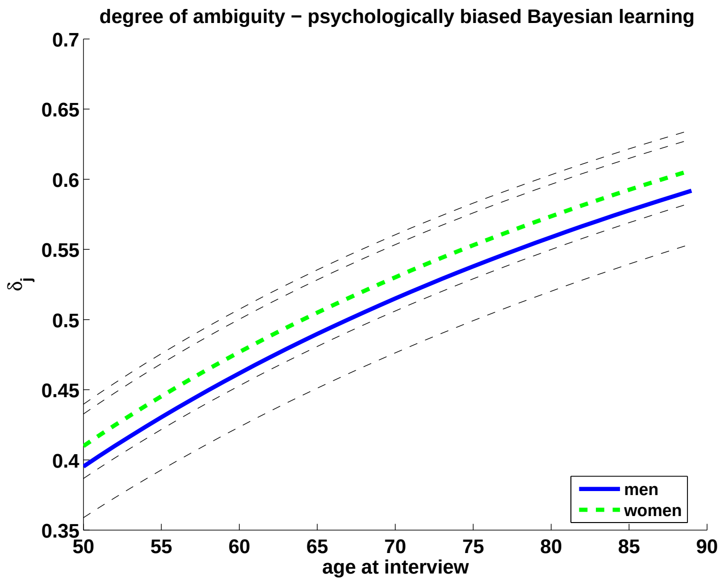


(b) Women



Notes: Black dashed lines are 95% confidence intervals obtained from 500 bootstrap iterations.
Source: Own calculations based on HRS, HMD and SSA data.

Figure 4: Degree of ambiguity (δ_h)



Notes: Black dashed lines are 95% confidence intervals obtained from 500 bootstrap iterations.
Source: Own calculations based on HRS, HMD and SSA data.

year of experience, cf. assumption 3, the factor of underestimation is given from equation (10) by $\frac{2\phi^{m-j}+h}{2+h} \cdot 100\% = \frac{2\phi^{30}+1}{3} \cdot 100\% = 35.4\%$ (36.2%). Finally, person at the age of 50 who has already gathered 31 years of experience ($h = 31$), underestimates the additive probability by factor $\frac{2\phi^{m-j}+h}{2+h} \cdot 100\% = \frac{2\phi^{30}+31}{33} \cdot 100\% = 94.1\%$ (94.2%) only.

We further find that the measure of optimism under ambiguity is significantly higher for men, i.e., $\lambda = 0.454$ with a 95% confidence interval of $[0.431, 0.476]$, than for women, i.e., $\lambda = 0.394$ with a 95% confidence interval of $[0.371, 0.419]$.[13] At the same time, the initial degree of ambiguity is almost identical for both sexes, also see figure 4 for the degree of ambiguity δ_h for all ages $50 - 89$. According to our interpretation of ambiguous beliefs, the weight $(1 - \delta_h)$ measures how much evidence gained from rational Bayesian learning is taken into account. Conversely, δ_h corresponds to the weight by which beliefs are affected by some "myside bias," in our model formalized as personal attitudes towards optimism, resp. pessimism, as measured by λ. A literal interpretation of our estimation results therefore suggests that respondents of both sexes are roughly affected by the same degree of ambiguity, but men resolve their ambiguity in a more optimistic manner than women. Furthermore, our results indicate that the initial ambiguity at the age of 19, cf. assumption 3, is rather low; the point estimates are about 0.02.

We next investigate the sensitivity of our results and the associated psychological interpretations with respect to focal point answers and the restrictions in assumption 3.

[13]The t-statistic of the two-sided t-test for equality of the point estimates is 3.53.

4.3 Sensitivity Analysis

Focal Point Answers

An apparently serious problem in data on subjective survival probabilities is the existence of "focal point answers" at self-reported survival probabilities of 0, 50, and 100 percent (Hurd and Kathleen 1995; Gan, Hurd, and McFadden 2003). One interpretation for individuals indicating probabilities of 0 or 100 percent is that they have not fully understood the question.[14] Thus, focal point answers could be regarded as implausible estimates of subjective probabilities. However, as discussed by Smith, Taylor, Sloan, Johnson, and Desvouges (2001) and Khwaja, Sloan, and Chung (2006), these focal point answers at 0% and 100% still have information content regarding the correct subjective belief because smokers provide the answer 0% more frequently than non-smokers. The target age-group specific answer pattern in our data displayed in figure 5 also illustrates that focal point answers have information content for the true subjective belief because the frequency of focal point answers at 0% increases with target age whereas the frequency of focal point answers at 100% decreases with target age. The overall pattern is the same for male and female respondents. Focal point answers at 50% may be due to round-off (Börsch-Supan 1998) or may reflect that respondents simply do not know much about their individual survival probability (Hurd, Rohwedder, and Winter 2005).

One approach to deal with these problems followed in the literature is to formally correct for focal point answers. Along this line, Gan, Hurd, and McFadden (2003) suggest a Bayesian procedure that reduces the distance between subjective survival curves and observed survival.[15] In our context, this approach is obviously meaningless because our aim is to explain the difference between subjective beliefs and the objective data. The alternative, followed by Smith, Taylor, and Sloan (2001), Smith, Taylor, Sloan, Johnson, and Desvouges (2001) and Khwaja, Sloan, and Chung (2006), is to acknowledge the information content of focal point answers and to examine the sensitivity of results with respect to these observations. We follow this latter approach.

In our model, focal point answers induce two sorts of biases. On the one hand, focal point answers at 0% and 100% bias the degree of pessimism observed at young ages and the degree of optimism observed at older ages, compare figure 1, downward and thereby towards the objective data. This is so because the focal point answer at 100% is primarily given by younger respondents whereas the focal point answer at 0% is primarily given by older respondents, compare figure 5. On the other hand, focal point answers at 50% induce opposite biases towards pessimism at younger ages and towards optimism at older ages. This is so because the objective survival probabilities of younger respondents are above 50% whereas those of older respondents are below 50%. Consequently, the first form of bias favors our interpretation of the data whereas the second form works against it.

[14] An alternative interpretation is that focal point answers reflect ambiguity, cf. Hill, Perry, and Willis (2004).

[15] Other researchers, such as Bloom, Canning, Moore, and Song (2006), correct for focal point answers by instrumental variables techniques.

We therefore next investigate the sensitivity of our results with respect to focal point answers by deleting in our sample all observations with focal point answers. This correction leaves us with a sample size of 24225 observations (10188 male and 14037 female respondents) for our sensitivity analysis, that is, roughly 46% of interviewees in our full sample have given focal point answers at either 0%, 50% or 100%, respectively.

Estimation results for this alternative data set are summarized in table 3. A comparison with our benchmark results in table 2 shows that the broad pattern of estimated parameter values does not change. The only discernable difference to our benchmark results is that the estimates of the degree of optimism, λ, do not differ much across sexes. Therefore, our earlier interpretation that men seem to resolve their ambiguity in a more optimistic manner than women, is not robust with respect to the exclusion of focal point answers.

Table 3: Parameter estimates – Excluding focal point answers

	$\hat{\Psi}$	$\hat{\sigma}(\Psi)$	$\widehat{CI}(\psi)$	$\hat{\Psi}$	$\hat{\sigma}(\Psi)$	$\widehat{CI}(\psi)$
Initial bias: ϕ	0.894	0.004	[0.886 0.900]	0.909	0.009	[0.901 0.934]
Degree of ambiguity: δ	0.023	0.003	[0.019 0.029]	0.028	0.002	[0.025 0.033]
Degree of optimism: λ	0.441	0.012	[0.417 0.467]	0.436	0.011	[0.415 0.455]
R^2	0.043	0.004	[0.035 0.051]	0.051	0.004	[0.043 0.058]
$\bar{R}^2$	0.826	0.055	[0.615 0.833]	0.879	0.033	[0.774 0.901]

Notes: These results are based on a sample which excludes focal point answers at 0%, 50% and 100%, respectively, cf. figure 5. $\hat{\Psi}$ are point estimates of model parameters, $\hat{\sigma}(\Psi)$ is the respective standard deviation and $\widehat{CI}(\Psi)$ is the respective 95% confidence interval. Standard errors are calculated by bootstrapping the subjective and objective survival probabilities by drawing with replacement in 500 bootstrap iterations.
Source: Own calculations based on HRS, SSA and HMD data.

Choice of Initial Age

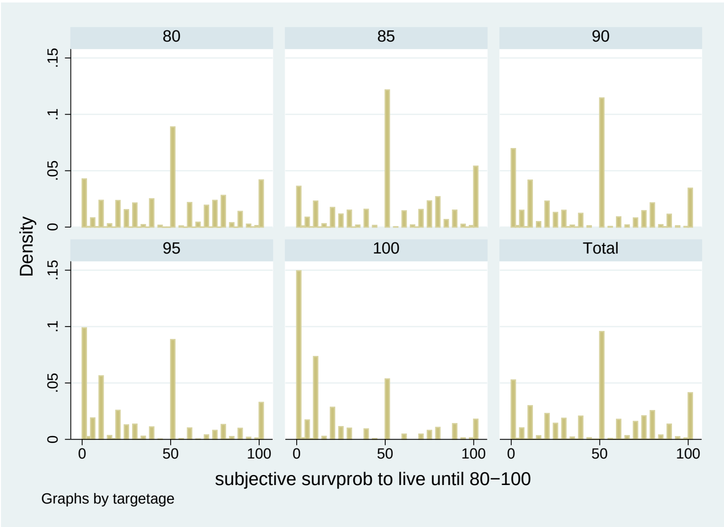
We next consider the sensitivity of our parameter estimates with respect to variations in the initial age of our model. In our benchmark results, the initial age was 20, cf. assumption 3, so that the youngest households observed in our data sample have already gathered $50-20+1 = 31$ years of experience. We here set the initial age to 50 so that the youngest household in our data sample and model has only gathered 1 year of experience. Results for this alternative specification are documented in table 4.

As documented in the table, the broad pattern of our results is not sensitive to our choice of the initial age. We however find that the point estimates of the initial bias and the initial degree of ambiguity are now significantly higher than in our baseline specification.[16]

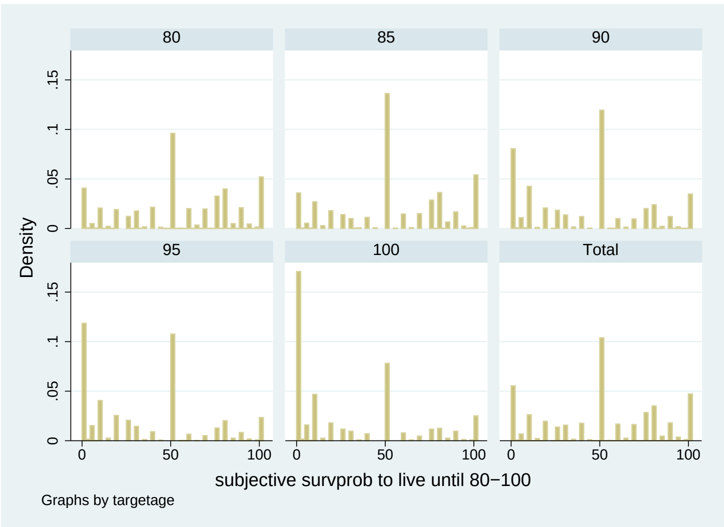
[16] The values of the t-statistics for the two-sided t-tests on equality of the respective coefficients across specifications are 16.5 (17.6) for ψ and 3.1 (8.5) for δ.

Figure 5: Answer pattern

(a) Men



(b) Women



Source: Own calculations based on HRS data.

Both results are rather mechanical effects due to our specification in proposition 2. First, observe from equation (10) that a decrease in the initial age decreases our index h and has to be compensated by an increase of ϕ in order to hold the overall bias for a given pair (j, m) constant. Second, observe from equation (9) that a decrease of experience (as measured by a decrease of the index h) has to be matched by an increase in δ to hold the degree of ambiguity for a given age, δ_h, constant.

Table 4: Parameter estimates – Sensitivity with respect to initial age

	Men			Women		
	$\hat{\Psi}$	$\hat{\sigma}(\Psi)$	$\widehat{CI}(\psi)$	$\hat{\Psi}$	$\hat{\sigma}(\Psi)$	$\widehat{CI}(\psi)$
Initial bias: ϕ	0.980	0.005	[0.968 0.990]	0.979	0.002	[0.974 0.983]
Degree of ambiguity: δ	0.034	0.004	[0.028 0.043]	0.048	0.003	[0.044 0.054]
Degree of optimism: λ	0.464	0.016	[0.432 0.494]	0.381	0.010	[0.361 0.403]
R^2	0.042	0.004	[0.034 0.049]	0.058	0.003	[0.052 0.064]
$\bar{R}^2$	0.819	0.035	[0.712 0.849]	0.900	0.014	[0.858 0.910]

Notes: These results are based on a specification of our model with an initial age of 50 rather than 20, cf. assumption 3. $\hat{\Psi}$ are point estimates of model parameters, $\hat{\sigma}(\Psi)$ is the respective standard deviation and $\widehat{CI}(\Psi)$ is the respective 95% confidence interval. Standard errors are calculated by bootstrapping the subjective and objective survival probabilities by drawing with replacement in 500 bootstrap iterations.
Source: Own calculations based on HRS, SSA and HMD data.

Speed of the Learning Process

Finally, we investigate the sensitivity of our results with respect to the speed of the learning Bayesian learning process that we restrict in assumption 3 to one. That is, we consider a specification in which the initial age is 20 as in our baseline results but the speed of the learning process is now ten times faster in that we assume $n(h) = 10 \cdot h$. Results for this specification that are reported in table 5 indicate that the speed of the learning process interacts with our estimate of the degree of ambiguity, δ, whereas the other parameters are roughly unaffected. More precisely, we find that, when the exogenous parameter ψ is ten times higher than in our baseline specification, then the estimate of parameter δ is about 10 times lower. Again, this mechanically follows from the specification of the learning model in proposition 2, especially equation (9).

Because of the simplicity of our model – owed to our wish for parsimony – we do not want to push the significance of any psychological interpretations of our findings too far. As a very robust result of our analysis, however, we first find for both genders that the assumption of a psychological "myside bias" in the interpretation of new information is required to explain the survival belief formation of a representative agent. Second, the initial degree of ambiguity required to generate these results, is relatively low because our

Table 5: Parameter estimates – Sensitivity with respect to ψ

	Men			Women		
	$\hat{\Psi}$	$\hat{\sigma}(\Psi)$	$\widehat{CI}(\psi)$	$\hat{\Psi}$	$\hat{\sigma}(\Psi)$	$\widehat{CI}(\psi)$
Initial bias: ϕ	0.893	0.002	[0.890 0.897]	0.901	0.002	[0.899 0.906]
Degree of ambiguity: δ	0.002	0.000	[0.002 0.003]	0.002	0.000	[0.002 0.003]
Degree of optimism: λ	0.438	0.011	[0.417 0.459]	0.386	0.011	[0.364 0.409]
R^2	0.039	0.003	[0.032 0.046]	0.062	0.003	[0.056 0.067]
$\bar{R}^2$	0.776	0.037	[0.662 0.812]	0.929	0.011	[0.890 0.931]

Notes: These results are based on a specification of our model with $n(h) = 10h$ rather than $n(h) = h$, cf. assumption 3. $\hat{\Psi}$ are point estimates of model parameters, $\hat{\sigma}(\Psi)$ is the respective standard deviation and $\widehat{CI}(\Psi)$ is the respective 95% confidence interval. Standard errors are calculated by bootstrapping the subjective and objective survival probabilities by drawing with replacement in 500 bootstrap iterations. *Source:* Own calculations based on HRS, SSA and HMD data.

estimates of δ range from $0.002 - 0.02$. This means that the initial deviation of our model from the standard rational Bayesian learning model is low.

5 Discussion

5.1 Selectivity

One criticism raised against using population averages as the relevant objective data is that our HRS sample may be prone to selectivity. Reasons for such selection biases are either that households have moved to nursing homes and are not followed by HRS interviewers or that sick people are reachable but may not be able to answer the questionnaire.[17] Such selection effects may explain (some of) the optimism we observe at higher ages in figure 1.

To address these concerns, we compute the HRS hazard rates between waves 2000 and 2002 and between waves 2002 and 2004, respectively, and compare them to the biannual mortality rates in the population for the respective years. In figure 6 we display the resulting hazard rates for men in panel (a) and for women in panel (b) between waves 2002 and 2004 for our full sample. The wiggles in the HRS data (dashed lines) are a consequence of relatively small sample size. Evidently, the HRS hazard rates correspond with the mortality rates in the population. The pattern is similar for the hazard rates between waves 2000 and 2002 (and also for our sample corrected by focal point answers)

[17] As Mike Hurd pointed out to us, the first selection effect was particularly severe for the early waves of the HRS because people were not followed into nursing homes in the past. Since we use the more recent waves of the HRS where people are in fact followed into nursing homes, selection effects may only play a role for the very old respondents in our sample, if at all.

and therefore not shown.[18]

5.2 Interpretation

Next to our interpretation of the data – pessimism at younger ages, respectively optimism at higher ages – several alternative interpretations are possible. First, the answering patterns depicted in figure 1 may reflect cohort rather than age effects. To accommodate this aspect we plotted the subjective data for various birth cohorts but this does not give indication for relevant cohort effects. These cohort plots are provided in appendix B.3. Second, the same answering patterns could be generated by a simple heuristic model with biases in subjective beliefs towards 0.5 which we shall refer to as a "0.5-bias".[19] Such a bias may, e.g., be due to the fact that individuals cannot deal with small probabilities of death, respectively of survival, irrespective of age. Consequently, all households underestimate objective survival probabilities that are above 0.5 (because they overestimate the relatively small probability of death) and overestimate objective survival probabilities that are below 0.5 (because they overestimate the relatively small probability of survival). Then, since the objective survival rates of the young are above 0.5 and of the old are below, this may drive the data. To us, our interpretation of the data is more sensible. First, pessimism at young age appears plausible because people may not accurately take into consideration future increases of life expectancy due to technological progress. Second, optimism at old age may result from the fact that people have survived the gamble against death several times before and thus develop an optimistic bias.[20]

To further support our interpretation of the data, figure 7 presents evidence from the German SAVE survey.[21] In this survey, individuals are first asked to provide an estimate for how long people of their cohort are going to live on average. Next, they are asked whether they expect to live as long as the average, longer or shorter. In contrast to questions for point estimates of survival probabilities, these are qualitative questions that are accordingly not prone to a potential "0.5-bias". As the graph in figure 7 illustrates, the fraction of households replying that they are expecting to live longer than average increases with age which clearly lends support to our interpretation of optimism at higher ages. Also notice that the gap between expecting to live longer vs. shorter than average increases after age $65-70$, just as the differences between the subjective and the objective survival rates in our HRS sample, cf. figure 1.

[18] If anything, we find for ages above 75 slightly higher mortality rates in the HRS between waves 2000 and 2002 than in the population which gives even more support to our interpretation of the data as "optimism" at higher ages.

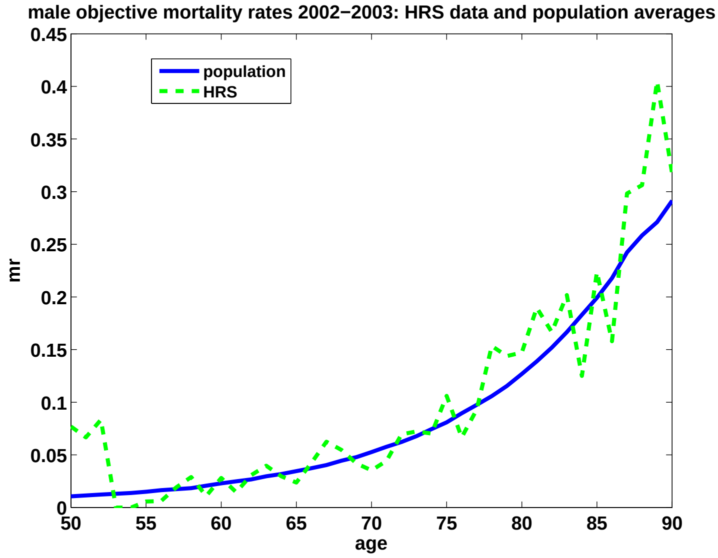
[19] This alternative explanation was pointed out to us by Mike Hurd.

[20] Dan McFadden made this point which favors our interpretation of the data.

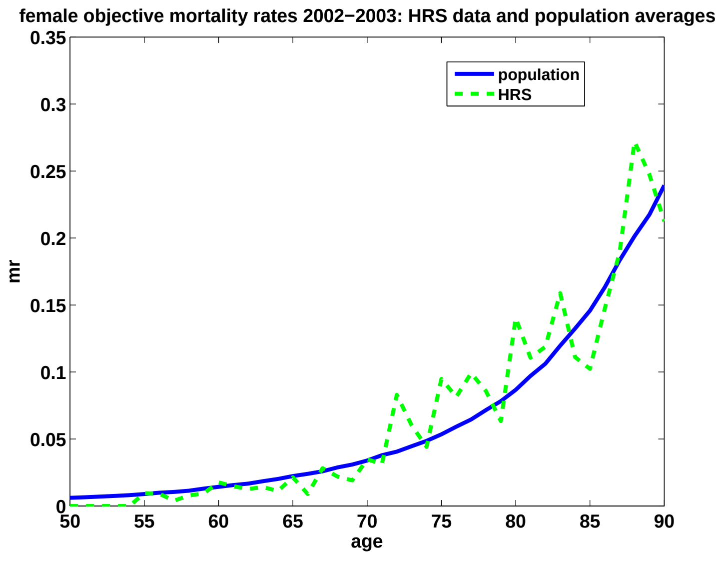
[21] SAVE is a panel study of German households that focuses on saving and investment behavior, cf. Schunk (2006) for a detailed description of the survey. We thank Bjarne Steffen for providing us with the data.

Figure 6: Objective survival rates in 2002-2003: HRS data versus population averages

(a) Men



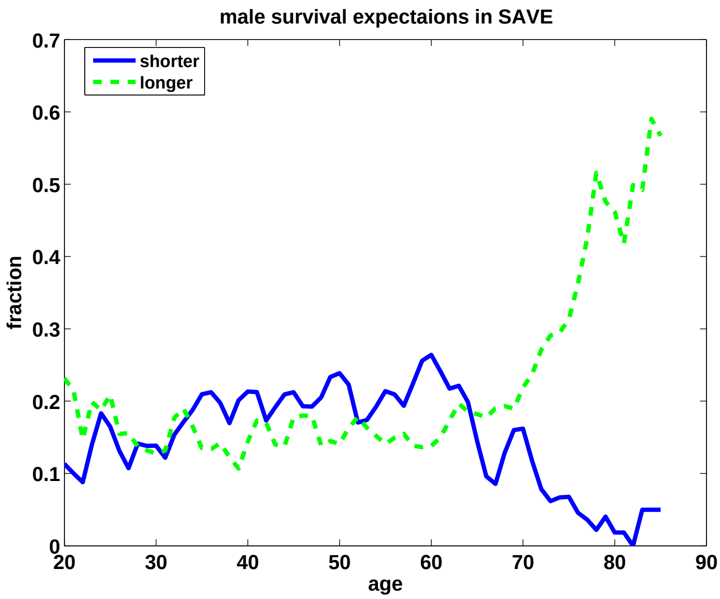
(b) Women



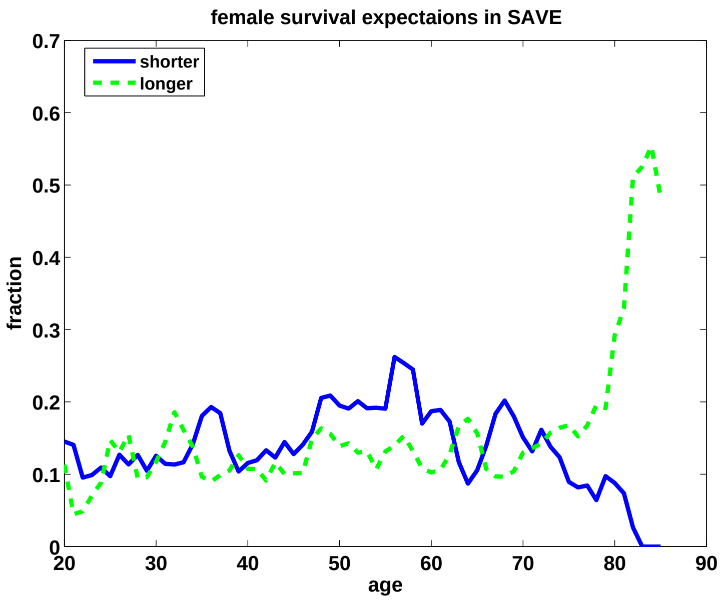
Notes: Solid line: population wide hazard rates (mortality rates) for 2002-2003. Dashed line: HRS hazard rates (mortality rates) between waves 2002 and 2004.
Source: Own calculations based on HRS, SSA and HMD data.

Figure 7: Subjective survival expectations in SAVE

(a) Men



(b) Women



Notes: Blue solid line: fraction of households expecting to live shorter than the average household of the respective age group. Green dashed line: fraction of households expecting to live longer than the average household of the respective age group.
Source: Own calculations based on SAVE 2005-2007, cf. Schunk (2006).

6 Conclusion

The HRS data on subjective survival beliefs suggest a violation of the rational expectations paradigm as well as of the rational Bayesian learning hypothesis. In a first step we therefore propose a new Choquet model of Bayesian learning that encompasses rational Bayesian learning while it additionally allows for the existence of a psychological bias in the interpretation of new information. For this purpose our formal approach combines concepts, such as non-additive beliefs and generalized Bayesian updating, from the theory of decision making under ambiguity with the standard approach of rational Bayesian updating. The resulting model of psychologically biased belief formation is very parsimonious in that it requires a low parametrization reflecting, first, an initially biased additive estimator of subjective survival probabilities, second, a measure for the agent's ambiguity with respect to her initial estimator of her subjective survival probability, and, third, a measure for the agent's optimistic versus pessimistic attitudes with respect to this ambiguity. Besides this parsimonious specification of the formation of subjective survival beliefs, our learning model has the additional advantages that, first, it is axiomatically founded within Choquet decision theory and, second, it is well supported by psychological evidence on diverging learning behavior.

In a second step we estimate the parameters of our Choquet model by pooling the HRS data. Despite the parsimonious parametrization we find that our model explains $80-94\%$ of the variation of average subjective survival probabilities in the data. The model's performance is statistically better for women than for men. For both genders we can clearly reject the hypothesis that the HRS data on subjective survival probabilities may be explained by rational Bayesian learning. The reason is that the rational Bayesian learning hypothesis implies convergence of the subjective probabilities to the respective objective data at higher ages but we instead observe an increasing degree of optimism in the data. On the contrary, our more sophisticated model of psychologically biased Bayesian learning can match these patterns in the data.

In our theoretical model we condition the updating of subjective beliefs on sex and age of individuals only by which we obtain a representative agent interpretation. We deliberately choose this strategy in order to focus the analysis on the main messages of this paper that Choquet Bayesian learning is a more appropriate model for survival belief formation than rational Bayesian learning. The strength of our parsimonious approach is certainly that we can directly map our model into life-cycle models of consumption and savings. Along this line, we will use our framework in future research in order to discuss the demand for annuities and to evaluate the implications of our model for life-cycle consumption and savings profiles.

However, our simple empirical strategy does not allow us to analyze how updating of beliefs differs across a variety of observed idiosyncratic health shocks or other events that are regarded as relevant for survival belief formation in the literature, such as parental death. In our future research, we plan to modify our theoretical model in such a way that the objective information is not based on average survival rates in the population but rather on objective information at the individual level. This would enable us to condition updating of beliefs on observed idiosyncratic shocks in between waves of the HRS, similar to Smith, Taylor, Sloan, Johnson, and Desvouges (2001). Along this line, our current research focuses on the characteristics of households who develop increasing optimism, respectively decreasing pessimism, in the SAVE survey, cf. figure 7.

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A Analytical Results

A.1 Generalized Bayesian Update Rule

In the present paper, we consider the so-called generalized (or full) Bayesian update rule. An axiomatic foundation under the assumption of CEU preferences is provided in Eichberger, Grant, and Kelsey (2006).

Definition 2. *The generalized Bayesian update rule for determining the conditional capacity* $\nu(A \mid B)$, $B \in \Sigma$, *for a given prior capacity* ν *is given as follows: for all* $A \in \Sigma$,

$$\nu(A \mid B) = \frac{\nu(A \cap B)}{\nu(A \cap B) + 1 - \nu(A \cup \neg B)}.$$

Observation 2. *An application of the generalized Bayesian update rule to a neo-additive prior results in the posterior belief*

$$\nu(A \mid B) = \delta_B \cdot \lambda + (1 - \delta_B) \cdot \mu(A \mid B) \tag{12}$$

whereby

$$\delta_B = \frac{\delta}{\delta + (1 - \delta) \cdot \mu(B)}. \tag{13}$$

Proof: Let $A, B \notin \{\emptyset, \Omega\}$ and $A \cap B \neq \varnothing$. Then

$$\begin{aligned}
\nu(A \mid B) &= \frac{\delta \cdot \lambda + (1 - \delta) \cdot \mu(A \cap B)}{\delta \cdot \lambda + (1 - \delta) \cdot \mu(A \cap B) + 1 - (\delta \cdot \lambda + (1 - \delta) \cdot \mu(A \cup \neg B))} \\
&= \frac{\delta \cdot \lambda + (1 - \delta) \cdot \mu(A \cap B)}{1 + (1 - \delta) \cdot (\mu(A \cap B) - \mu(A \cup \neg B))} \\
&= \frac{\delta \cdot \lambda + (1 - \delta) \cdot \mu(A \cap B)}{1 + (1 - \delta) \cdot (\mu(A \cap B) - \mu(A) - \mu(\neg B) + \mu(A \cap \neg B))} \\
&= \frac{\delta \cdot \lambda + (1 - \delta) \cdot \mu(A \cap B)}{1 + (1 - \delta) \cdot (-\mu(\neg B))} \\
&= \frac{\delta \cdot \lambda + (1 - \delta) \cdot \mu(A \cap B)}{\delta + (1 - \delta) \cdot \mu(B)} \\
&= \delta_B \cdot \lambda + (1 - \delta_B) \cdot \mu(A \mid B)
\end{aligned}$$

with δ_B given by (13).

A.2 Unconditional Probability of Receiving Information $\mathbf{I}_n^k$

We have that

$$
\begin{aligned}
\mu\left(\mathbf{I}_n^k\right) &= \frac{\mu\left(\mathbf{I}_n^k \mid \pi\right)\mu\left(\pi\right)}{\mu\left(\pi \mid \mathbf{I}_n^k\right)} \\
&= \frac{\binom{n}{k}\pi^k\left(1-\pi\right)^{n-k}\mu\left(\pi\right)\cdot K_{\alpha,\beta}\pi^{\alpha-1}\left(1-\pi\right)^{\beta-1}}{K_{\alpha+k,\beta+n-k}\pi^{\alpha+k-1}\left(1-\pi\right)^{\beta+n-k-1}} \\
&= \binom{n}{k}\frac{K_{\alpha,\beta}}{K_{\alpha+k,\beta+n-k}} \\
&= \binom{n}{k}\frac{\Gamma\left(\alpha+\beta\right)}{\Gamma\left(\alpha\right)\Gamma\left(\beta\right)}\frac{\Gamma\left(\alpha+k\right)\Gamma\left(\beta+n-k\right)}{\Gamma\left(\alpha+\beta+n\right)} \\
&= \binom{n}{k}\frac{\Gamma\left(\alpha+\beta\right)}{\Gamma\left(\alpha+\beta+n\right)}\cdot\frac{\Gamma\left(\alpha+k\right)}{\Gamma\left(\alpha\right)}\cdot\frac{\Gamma\left(\beta+n-k\right)}{\Gamma\left(\beta\right)} \\
&= \binom{n}{k}\frac{\Gamma\left(\alpha+\beta\right)}{\left(\alpha+\beta+n-1\right)\cdot\ldots\left(\alpha+\beta\right)\cdot\Gamma\left(\alpha+\beta\right)} \\
&\quad\cdot\frac{\Gamma\left(\alpha+k-1\right)\cdot\ldots\cdot\alpha\cdot\Gamma\left(\alpha\right)}{\Gamma\left(\alpha\right)} \\
&\quad\cdot\frac{\Gamma\left(\beta+n-k-1\right)\cdot\ldots\cdot\beta\cdot\Gamma\left(\beta\right)}{\Gamma\left(\beta\right)} \\
&= \binom{n}{k}\frac{\left(\alpha+k-1\right)\cdot\ldots\cdot\alpha\cdot\left(\beta+n-k-1\right)\cdot\ldots\cdot\beta}{\left(\alpha+\beta+n-1\right)\cdot\ldots\cdot\left(\alpha+\beta\right)}
\end{aligned}
$$

whereby the last equality readily follows from the fact that $\Gamma\left(x\right)=\left(x-1\right)\cdot\Gamma\left(x-1\right)$ for $x>1$ (Rudin 1976, Theorem 8.18).

B Data

According to our model two different types of data are required for the empirical analysis that follows in section 4: (i) subjective conditional beliefs to live until target age and (ii) predicted objective conditional probabilities to live from age r to age $r+1$ for all $r=j,\ldots,m-1$. We here describe our data sources and the methodologies we apply to construct these data.

B.1 HRS Data

The HRS is a national representative panel survey of individuals aged 50 and older and their spouses. In addition to respondents from eligible birth years, the survey interviewed the spouses or partners of the respondents, regardless of age. Thus, some (mostly female) individuals are younger than 50 and few, younger than 40. In our application we focus on the target group of the HRS and therefore only look at individuals of age 50 and older.

Some respondents of the above question were 90 years old at the time of interview. We do not include these observations in our analysis.

Younger HRS interviewees were also asked about their probabilities to live until age 75. Some of these respondents have given inconsistent answers at certain points of time as their self-reported probabilities to live until 75 are lower than the self-reported probabilities to live until 80 or 85. We excluded these cases of evidently inconsistent answering patterns. Furthermore, in some cases, individuals reported the same probability to live until age 75 as to live until age 80 or 85. As this answering pattern may be due to pure rounding and is not strictly inconsistent with our theoretical model, we keep these observations in the sample.

The presence of focal point answers in our data is discussed in subsection 4.3.

B.2 Cohort Life Tables

We adopt the Lee-Carter procedure (Lee and Carter 1992) to estimate trends in mortality and to project survival rates into the future. The procedure allows us to describe and to project the development of age-specific mortality rates over time within a parsimonious framework. Basically, the model splits mortality rates into age-specific components that are constant over time and a time varying survival index capturing the development of mortality. Then, one can extrapolate the time series of the mortality index by means of a suitable time series model. Future age-specific mortality rates can be recovered by linking the projected mortality index to the age-specific components.

To describe the methodology, we now introduce a time index t. Following Lee and Carter (1992) we decompose the average objective age-specific survival probability in period t as

$$\log(\pi^{\star}_{t,r,r+1}) = a_r + b_r d_t \tag{14}$$

where a_r and b_r are the age-specific constants and where d_t is the time specific factor. We opt for a parsimonious representation of the time series process of d_t and assume that d_t follows a unit root process with drift

$$d_t = \theta + d_{t-1} + \epsilon_t. \tag{15}$$

where $\epsilon \sim \mathcal{N}(0, \sigma^2_\epsilon)$.

We assign objective survival probabilities to each respondent in our HRS panel in each wave $\tau \in \{2000, 2002, 2004\}$ as follows. We estimate for each wave τ, sex specific values of $\hat{a}_r$, $\hat{b}_r$, $\hat{\theta}$, $\hat{\sigma}_\epsilon$ and calculate predicted values of $\hat{\pi}^{\star}_{t,r,r+1}$ using data only until period τ. We then proceed to the next wave and update the objective information also using the data for the two years in between periods τ and $\tau + 2$. Our predictions of future objective survival probabilities, $\hat{\pi}^{\star}_{t,r,r+1}$, are calculated by iterating forward on

$$\hat{d}_t = \hat{\theta} + \hat{d}_{t-1} \tag{16}$$

and

$$\hat{\pi}^{\star}_{t,r,r+1} = \exp\left(\hat{a}_r + \hat{b}_r \hat{k}_t\right). \tag{17}$$

While we ignore uncertainty of our estimates of the age-vectors a_r and b_r, we account for uncertainty of the objective data by calculating standard deviations and confidence intervals of $\hat{\theta}$ by bootstrapping. This uncertainty is also reflected in our estimates reported in section 4. Table 6 reports the sex and wave specific point estimates $\hat{\theta}$ and the respective standard deviations. Estimated parameter values for waves 1, 2 and 3 are based on population data from HMD and SSA for $1900-2000$, $1900-2002$ and $1900-2004$, respectively.

Table 6: Parameter estimates of $\hat{\theta}$

	Men		Women	
	$\hat{\theta}$	$\hat{\sigma}(\theta)$	$\hat{\theta}$	$\hat{\sigma}(\theta)$
wave 1	-1.4186	0.5336	-1.8586	0.5339
wave 2	-1.4123	0.5426	-1.8287	0.5336
wave 3	-1.4518	0.4927	-1.8462	0.5103

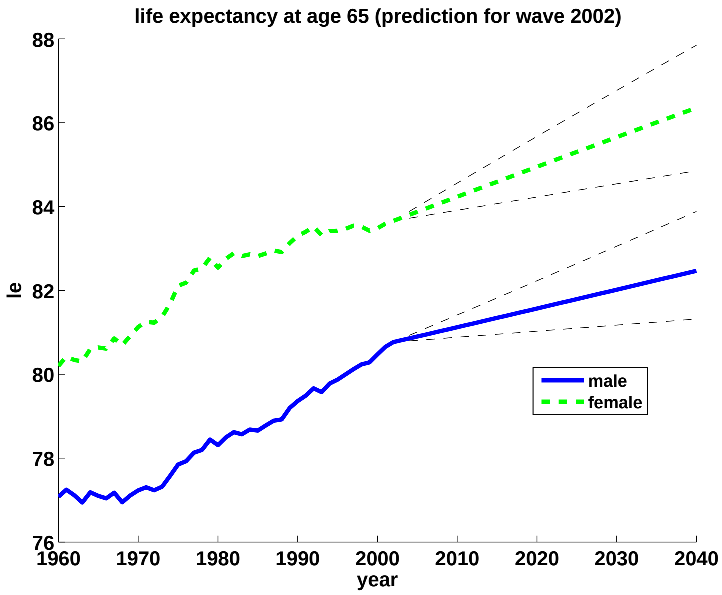
Notes: Standard errors of $\hat{\theta}$ are calculated from 500 bootstrap iterations.
Source: Own calculations based on SSA and HMD.

Figure 8 shows data on, and predicted values for, the remaining life expectancy at age 65 for wave 2002. The dashed lines are the bootstrapped 95% confidence intervals. The new information on objective survival probabilities between waves only leads to small changes in these predictions. Results for other years are therefore not shown. Furthermore, life expectancy at birth and the remaining life-expectancies at other ages display similar trends whereby the trend is increasing with age.

B.3 Cohort Effects

To accommodate the criticism that cohort effects may drive the pattern displayed in figure 1, figure 9 presents the subjective beliefs for various cohorts. As there are no clear-cut gaps between the respective line segments that represent birth cohorts, this stylized evidence can not be regarded as an indication for relevant cohort effects.

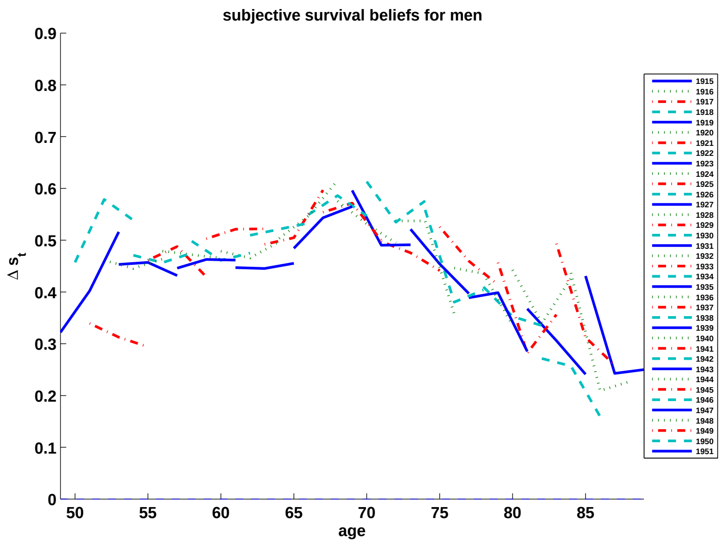
Figure 8: Predicted life expectancy at age 65 in year 2002



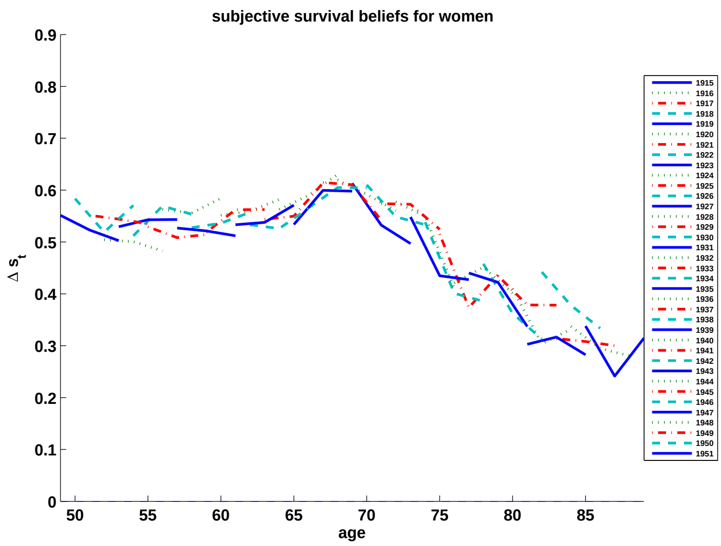
Notes: Black dashed lines are 95% confidence intervals obtained from 500 bootstrap iterations.
Source: Own calculations based on HMD and SSA data.

Figure 9: Subjective survival expectations by cohorts

(a) Men



(b) Women



Notes: These graphs display the subjective beliefs of figure 1 for various birth cohorts.
Source: Own calculations based on HRS.

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