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Executive Summary

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RETHINKING OPTIMAL WEALTH ACCUMULATION AND DECUMULATION STRATEGIES IN THE WAKE OF THE FINANCIAL CRISIS

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A substantial literature analyzing models of optimal savings, asset allocation, drawdown, and annuitization is based on the assumption that stock returns are normally distributed with a constant mean. But other research suggests that returns appear to oscillate around their long-run average in persistent waves and that the distribution of returns, particularly for stocks, has fat tails and might be skewed.

Using annual data for stock and bond returns from 1926 through 2011, this paper first investigates the distribution of historic returns and then uses that distribution to determine optimal consumption and portfolio asset allocation for a risk-averse household facing labor-income uncertainty and longevity risk. The household is also entitled to Social Security retirement benefits.¹

Although the data contain a higher incidence of large negative returns than permitted by the normal distribution, statistical tests do not permit us to reject the null hypothesis that returns are normally distributed or the null hypothesis that returns follow a stable distribution with fatter tails.

This paper considers optimal consumption and asset allocation, given three alternative assumptions about the distributions of stock and bond returns: 1) returns are normally distributed with a constant mean; 2) returns follow a vector-autoregressive (VAR) process so that after a set of large negative returns, future returns recover, with the new path for the expected value of assets tending to run below the expectations established before the disturbances occurred; disturbances are normally distributed; and 3) returns follow a VAR process, with disturbances drawn from a stable distribution that is skewed and has fat tails.

The distribution of returns in any year depends on the path of prior returns. Consequently, solving the model with recursive dynamic programming methods is prohibitive.² This project therefore proceeds as follows. First, a joint distribution of disturbances is approximated with a Monte Carlo sample. It draws

¹ The household is assumed to have constant relative risk aversion utility, with a coefficient of risk aversion of two or five. Following Scholz, Seshadri, and Khitatrakun (2006), labor market earnings are assumed to follow an AR(1) process.

² Numerical techniques also require that continuous distributions be discretized. Unless an infeasibly large number of grid points is used, it is not possible to capture skewness and fat tails.

100,000 paths of disturbances for earnings, the vector of returns for stocks and bonds, and for mortality starting at age 20 and for 81 periods, up to an assumed maximum survival age. The optimal plan for consuming earnings and allocating wealth maximizes expected utility over the domain defined by these consumption and asset allocation paths. We approximate the household's plan for consumption and for allocating its wealth using six parameters. The optimal plan is conditional on the information it possesses at age 20. As it ages, it recasts its plan and makes mid-course corrections as it gains information about its earnings and returns.

Figure 1 shows optimal stock allocations, by age, for each of the three assumed distributions of stock returns, assuming that the household is relatively risk-averse, with a coefficient of risk aversion of five.

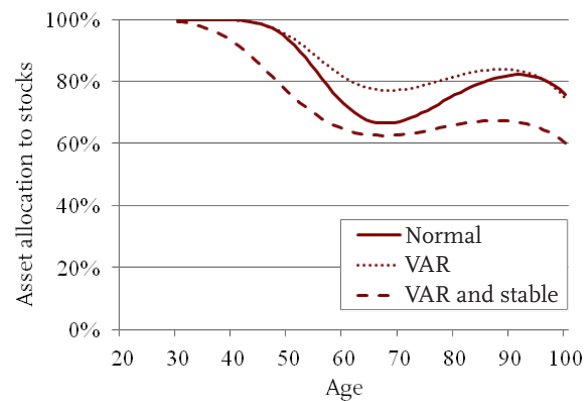
When returns are assumed to be normally distributed with a constant mean, the optimal stock allocation declines with age (Jagannathan and Kocherlakota 1996), reflecting the declining value of the household's low-risk human capital. At advanced ages, there is a slight increase in the optimal stock allocation, because at this point in the life cycle almost all of the household's wealth is in risk-free Social Security, a close substitute for bonds.

When returns are assumed to follow a VAR process with normally distributed disturbances, stocks are less risky, because periods of poor returns are likely to be followed by periods of above average returns. Households optimally choose to invest larger proportions of their financial assets in stocks at all ages, reflecting the lower risk. When returns follow a VAR with disturbances drawn from a stable distribution, meaning that returns partially recover with returns drawn from a stable distribution, the optimal portfolio allocation to stocks is somewhat less than when disturbances are normally distributed. Although the household benefits from the recovery in returns following a period of poor returns, this is approximately offset by the additional risk the household faces as a result of the disturbances being drawn from a stable distribution.

Finally, the model is used to calculate the impact on household financial well-being of basing consumption and asset allocation decisions on incorrect beliefs about the stochastic process governing returns. The metric is asset allocation equivalent consumption (AAEC), which is defined as the percent increase in per-period consumption that would leave the household indifferent between 1) receiving a consumption "bonus" and basing its consumption and asset allocation decisions on an incorrect model, and 2) receiving no bonus and basing its decisions on the correct model.

Assuming constant relative risk-aversion and a coefficient of risk aversion of two, AAEC is 0.95 percent if the household incorrectly believes returns are drawn from a normal distribution with a constant mean, when in fact they follow a VAR with disturbances drawn from a stable distribution. AAEC is 1.17 percent when the household incorrectly believes returns follow a VAR with normal disturbances. When the coefficient of risk-aversion is five, AAEC is 0.03 and 0.07 percent, respectively, reflecting the much lower accumulation of wealth and the lower allocation of assets to stocks by risk-averse households.

FIGURE 1. OPTIMAL ASSET ALLOCATION TO STOCKS, ASSUMING HOUSEHOLD RISK AVERSION OF 5.



Source: Authors' calculations.

The majority of households derive most of their lifetime consumption from labor market earnings and Social Security, with earnings from financial assets being relatively insignificant. Unless households plan to accumulate a substantial amount of financial assets and invest a substantial share of these assets in stocks, their incorrect portfolio allocations reduce their lifetime utility and lifetime consumption relatively little.

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