

# WHAT DOES IT COST TO GUARANTEE RETURNS?

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## Introduction

The financial crisis has dramatically demonstrated how a collapse in equity prices can decimate retirement accounts. The crisis highlights the fragility of existing 401(k) plans as the only supplement to Social Security and has sparked proposals to reform the retirement income system. One component of such a system could be a new tier of retirement accounts. Given the declines in the share of earnings Social Security will replace, these accounts would bolster replacement rates for low-wage workers and increase the security of middle- and upper-wage workers who increasingly rely on their 401(k) plans to supplement Social Security. However, these new accounts could face the same risk of collapse in value seen over the past year in 401(k)s. So policymakers may find some form of guaranteed return or risk sharing desirable to prevent huge variations in outcomes.<sup>1</sup> This *brief* explores the feasible range and the cost of the first option – guarantees.

This *brief* is structured as follows. The first section reviews the argument for more retirement saving and shows the inevitable volatility that results from leaving the outcomes completely up to the market. The second section shows that, *in retrospect*, it would have

been quite cheap to have guaranteed relatively high real rates of return on individual account balances and that only high guarantees would have smoothed returns across cohorts in a meaningful way. The third section uses finance theory to price guarantees *prospectively*, finding that guarantees in excess of the risk-free rate are not possible if the guarantor shares the market's aversion to risk. The fourth section concludes that, as long as the guarantor shares the market's aversion to risk, rate of return guarantees are unlikely to solve the problem of wide variations in outcomes due to market fluctuations. Guaranteeing the riskless rate would have had no impact historically. And finance theory suggests that insurers cannot guarantee returns greater than the riskless rate unless they are willing and able to bear more risk than other investors.

## The Need for More Retirement Income

People need more retirement saving because the existing retirement income system is contracting

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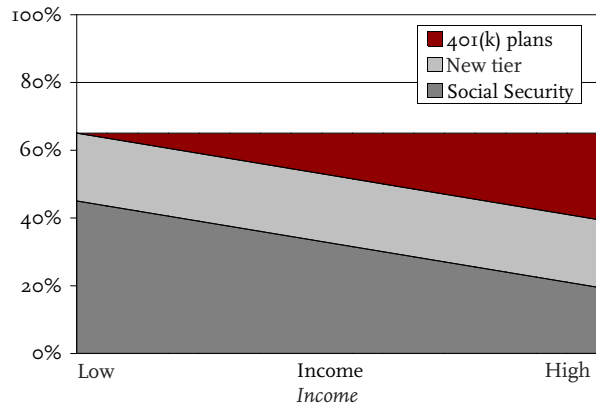
and people are living longer. At any given retirement age, Social Security benefits will replace a smaller fraction of pre-retirement earnings than in the past. First, the increase in the Full Retirement Age from 65 to 67 is equivalent to an across-the-board cut. Second, the taxation of Social Security benefits under the personal income tax will move further down the income distribution, as the exemption amounts in the tax code are not indexed to wage growth or inflation. Additional benefit cuts that might be enacted to shore up the solvency of the Social Security program would further lower replacement rates.

With a diminished role for Social Security, retirees will be increasingly dependent on employer-sponsored pensions. At any moment in time, however, less than half of the private sector workforce age 25-64 participates in any employer-sponsored plan. And those who do have employer-sponsored coverage find themselves increasingly reliant on 401(k) plans. In theory, workers could accumulate substantial wealth in a 401(k), but the Federal Reserve's 2004 *Survey of Consumer Finances* reports that the typical household head approaching retirement (age 55-64) had 401(k)/IRA balances of only \$60,000.<sup>2</sup> Although 401(k) plans received a boost from the Pension Protection Act of 2006, which encouraged employers to make their plans easier and more automatic, the basic fragility of 401(k)s was exposed by the current financial crisis, which has reduced the value of equities in 401(k)s/IRAs by about \$2 trillion.<sup>3</sup>

Given the decline in Social Security and employer-provided pensions, workers could save more on their own. But they have not. Thus, many future retirees – both those who must rely only on Social Security and those who have a supplementary 401(k) plan – are likely to have inadequate retirement income. Proposals to expand coverage through automatic IRAs or a universal 401(k) implicitly claim that those who already have a supplementary plan will be adequately prepared for retirement. As indicated above, this assumption is not correct. Thus, the vast majority of future retirees will need an additional tier of retirement saving. Figure 1 presents a schematic of what an additional tier might look like.

An earlier *brief* showed that replacement rates – benefits as a percent of pre-retirement earnings – produced by a defined contribution account will vary dramatically depending on the performance of the stock market during the period over which the participant is working and accumulating assets (see Figure

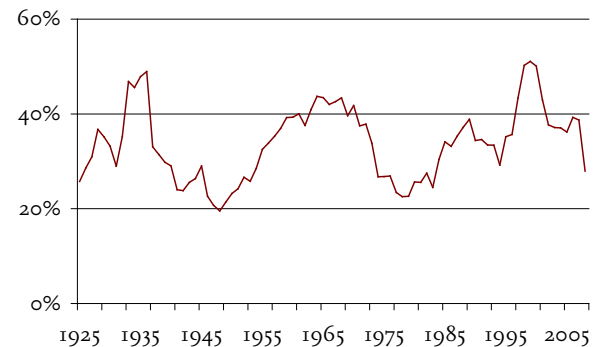
FIGURE 1. ADDITIONAL TIER OF FUNDED, PRIVATELY-MANAGED RETIREMENT SAVING



Source: Authors' illustration.

2).<sup>4</sup> This pattern occurs even when individuals invest in a target date fund. These accumulations are only somewhat offset by variations in interest rates at retirement when individuals are assumed to use their accumulations to purchase a real (inflation-adjusted) annuity.

FIGURE 2. REPLACEMENT RATE FROM A REAL ANNUITY BASED ON ACCUMULATIONS IN A TARGET DATE FUND



Source: Munnell, Webb, and Golub-Sass (2008).

The recent financial crisis highlights another problem with defined contribution accounts – namely, values can drop precipitously just as participants are approaching retirement. Sharp drops in retirement balances upset people's plans even if the drops merely offset a lifetime of high returns. Earlier high returns are likely to have encouraged people to cut back on their saving, sensing they had "enough" for retirement. As a result, a market collapse leaves most people with inadequate retirement saving.

The question is whether policymakers would view as acceptable such widely different replacement rates and the potential for large declines. If such outcomes are not acceptable, mechanisms would be needed to address these problems in the new tier of retirement saving. Interestingly, the two problems are intertwined, and solving one substantially ameliorates the other, as will be shown below.

## Guarantees in Retrospect

The most straightforward approach would be to guarantee rates of return.<sup>5</sup> The following exercise estimates when and how much the guarantor would have had to pay out to cover different levels of guarantees. The model assumes that workers enter the workforce at 22, work steadily for 43 years and retire at age 65, and enjoy real wage growth of 2 percent a year. Each year from 1883 to 2008 workers contribute 4 percent of their income to their account and invest their contributions in a fund of U.S. equities. Over that period, the stock market returned 7.6 percent after inflation with a standard deviation of 19.5 percent. In the analysis below, the first cohort reaches age 65 in 1925.

Although we are ultimately interested in replacement rates rather than age 65 accumulations, to simplify the exercise we ignore the effect of interest rates on annuity prices and focus on accumulations at age 65. Thus, the question becomes how often and how much would a guarantor have had to pay in order to provide workers when they reached age 65 a real return of 2 percent, 3 percent, 4 percent, 5 percent, or 6 percent on their lifetime contributions. To satisfy each of these guarantees, Table 1 shows the dates and the amounts that would have had to be transferred to those age 65 as a percent of Gross Domestic Product (GDP). Note that the table shows only those years that involve a required payment. See the Appendix for additional details on the calculations.

The results reveal, *based on historical data*, that a 2-percent and 3-percent guarantee would never have required any payments. The reason is that, over the period under consideration, a portfolio fully invested in equities never yielded less than 3.8 percent averaged over an individual's work life. A 4-percent guarantee would have required payments in three years out of the 84 years; a 5-percent guarantee would have required payments in eight years; and a 6-percent guarantee would have required payments in 27 out of

the 84 years. For example, with a 6-percent guarantee, the guarantor would have had to pay out to those turning 65 in 2008 an amount equal to 0.42 percent of GDP, about \$60 billion.

TABLE 1. REQUIRED GUARANTEE PAYMENT AS A PERCENT OF GDP BY GUARANTEED RATE OF RETURN

Year	Guaranteed Rate of Return				
	2%	3%	4%	5%	6%
1925	-	-	-	-	0.59%
1926	-	-	-	-	0.52%
1931	-	-	0.06%	0.82%	1.85%
1932	-	-	0.17%	1.19%	2.55%
1933	-	-	-	-	1.04%
1934	-	-	-	0.05%	1.39%
1937	-	-	-	-	1.01%
1939	-	-	-	-	0.21%
1940	-	-	-	-	0.51%
1941	-	-	0.01%	0.32%	0.74%
1942	-	-	-	0.23%	0.61%
1943	-	-	-	0.04%	0.38%
1944	-	-	-	-	0.25%
1946	-	-	-	-	0.31%
1947	-	-	-	0.05%	0.39%
1948	-	-	-	0.07%	0.41%
1949	-	-	-	-	0.28%
1974	-	-	-	-	0.36%
1977	-	-	-	-	0.10%
1978	-	-	-	-	0.28%
1979	-	-	-	-	0.28%
1980	-	-	-	-	0.07%
1981	-	-	-	-	0.40%
1982	-	-	-	-	0.26%
1983	-	-	-	-	0.06%
1984	-	-	-	-	0.14%
2008	-	-	-	-	0.42%

Source: Authors' calculations based on U.S. Bureau of Economic Analysis (2008); Officer and Williamson (2008); U.S. Bureau of Labor Statistics, *Current Population Survey* (CPS) (1962-2008); and U.S. Census Bureau, *Decennial Census* (1920-1960).

Two key questions regarding guarantees in retrospect are how much they would have cost and whether they would have done the job of smoothing replacement rates and avoiding major upset to people's plans. The following discussion assumes that the guarantor is the government.<sup>6</sup> Relying on the private sector for even low levels of guarantees raises issues relating to the continuity of the insurer and the availability of a natural hedge. Given the recent demise of Bear Stearns and Lehman Brothers and the plight of AIG, individuals would have no confidence that the firm offering the guarantee would be there for the payoff forty years down the road. And private sector firms would have no natural hedge to insure against the possibility of having to cover the guarantee, since very few counterparties exist that would gain from a sharp economic downturn. Thus, the government becomes the only realistic source of guarantees and the questions are whether the government *in retrospect* could have afforded the cost of guarantees and whether guarantees would have smoothed fluctuations.

### Cost of Guarantees in Retrospect

The cost depends on what happens to returns in excess of the government guarantee. That is, assume the government guarantees a 4-percent return, but those turning 65 in a given year had earned 8 percent over their worklives. The additional 4 percent could accrue to the individuals – that is, they get the upside – or it could go to the government to offset future bad years. Over the past 84-year period, if the government had gotten the whole upside, it would have experienced no net cost – even at a guarantee level of 6 percent. In fact, the government would have made money (see Table 2).

TABLE 2. COST OF GOVERNMENT GUARANTEES AS A PERCENT OF GDP, DEPENDING ON ENTITY KEEPING THE UPSIDE, 1925-2008

Guaranteed rate of return	Entity keeping the upside	
	Government	Individuals
3 percent	-2.194%	0.000%
4 percent	-1.864	0.003
5 percent	-1.424	0.033
6 percent	-0.834	0.183

Source: Authors' calculations based on U.S. Bureau of Economic Analysis (2008); Officer and Williamson (2008); 1962-2008 CPS; and 1920-1960 *Decennial Census*.

On the other hand, if individuals had kept the whole upside, the government would have faced some costs (see Table 2). The government could have issued debt when the payments were made and spread the costs over, say, the next 30 years. On average, the government would have needed to raise taxes by an amount equal to 0.18 percent of GDP (roughly \$26 billion in 2008). Alternatively, the costs could have been covered by increasing the individual contribution rate. For example, to guarantee each individual retiring at age 65 an annual average rate of return of 6 percent, the contribution rate would have had to be raised from 4 percent to 4.36 percent. The basic conclusion is that regardless of how costs are measured, guarantees – even high levels of guarantees – would have been totally affordable in retrospect.

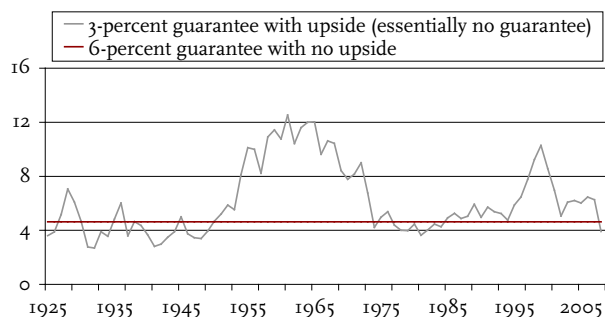
### Effect of Guarantees in Retrospect

The impetus for the discussion of guarantees was twofold. First, policymakers might not want a government-sponsored tier of retirement income to have replacement rates varying from 20 percent to 50 percent. Second, it might be desirable to avoid a system where participants can see their account balances drop by 30 percent as they approach retirement. The question is the extent to which guarantees solve these problems.

*Smoothing replacement rates.* Providing low levels of guarantees – say 2 or 3 percent – and allowing individuals to keep the upside, does virtually nothing to eliminate the fluctuations. As discussed above, no group turning 65 in the last 84 years would have seen a lifetime return of less than 3.8 percent. Therefore, the guarantee would never have been paid, and the pattern of accumulations relative to final earnings would have been identical to that produced by the fluctuations in the market (see Figure 3 on the next page). On the other hand, a guarantee of, say, 6 percent, with the upside going to the government, stabilizes accumulations relative to final earnings by providing everyone with the same amount.

The stability in accumulations relative to earnings appears to be purchased at the expense of some groups of retirees foregoing substantial gains. This outcome is the result of picking a guarantee of 6 percent when equities have yielded 7.6 percent. A higher guarantee would eliminate the apparent problem, but would put future taxpayers on the hook for bigger payments. But it is worth noting that even a guarantee of 6 percent allows individuals to invest all their

FIGURE 3. RATIO OF ASSETS TO FINAL SALARY, ASSUMING A 3-PERCENT GUARANTEE WITH UPSIDE AND A 6-PERCENT GUARANTEE WITH NO UPSIDE

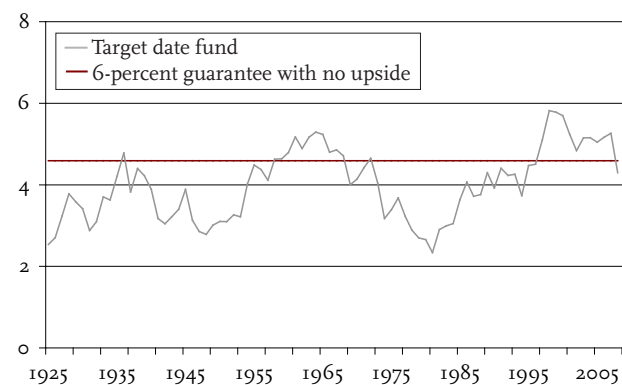


Source: Authors' calculations based on Burtless (2000); and Burtless (2008).

contributions in the new tier in equities. Without a guarantee, most financial experts would advise individuals to invest in a suitable target date fund where the percent in equities declines as the person ages. Such an approach would produce not only wide variations in accumulations, as discussed earlier, but also lower accumulations than a 6-percent return (see Figure 4).

*Avoiding sharp drops.* As in the case of smoothing, a guarantee of 2 or 3 percent would have had no effect, while a guarantee of 6 percent would have avoided a

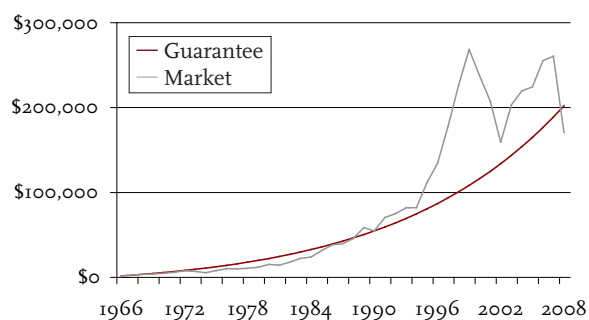
FIGURE 4. RATIO OF ASSETS TO FINAL SALARY, ASSUMING A 6-PERCENT GUARANTEE WITH NO UPSIDE AND A TARGET DATE FUND



Source: Authors' calculations based on Burtless (2000); Burtless (2008); and Fidelity Investments (2008).

major calamity in 2008. Figure 5 shows the accumulated balances in a defined contribution account for a hypothetical individual entering the labor force in 1966 at age 22 and turning 65 in 2008 under a 6-percent guarantee and under market returns.<sup>7</sup> Under a 6-percent guarantee, where the government gets the upside, the individual would have been entitled to \$188,000 in 2007, while the market value of assets in the account would have been \$259,600. In 2008, the financial collapse would have reduced the market value of assets by 35 percent to \$169,600. But, with the guarantee of a cumulative lifetime return of 6 percent, the government would have transferred \$31,500 to the individual's guaranteed account. This transfer would have brought the total to \$201,100 – producing a predictable lifetime guarantee of 6 percent on accumulated contributions.

FIGURE 5. GUARANTEED ACCOUNT BALANCE ASSUMING A 6-PERCENT RETURN AND MARKET VALUES



Year	Guarantee	Market
2007	\$188,000	\$259,600
2008	\$201,100	\$169,600

Source: Authors' calculations.

## Prospective Guarantees

The past 83 years represent only a limited number of draws from an unlimited number of outcomes that could have occurred, and therefore cannot be used to determine the potential cost of guarantees going forward. Instead, standard finance theory allows us to estimate the market price of future guarantees in a manner that reflects both the risk of insurers' experiencing substantial shortfalls and their aversion to bearing these risks.

A guaranteed minimum average rate of return on pension balances invested in stocks is a kind of put option, commonly known as a floor, which commits insurers to top up retirees' pension accounts when their average returns fall short of the guaranteed return. The price of a floor can be estimated assuming that insurers' aversion to risk matches the market average and that, in the future, equity returns follow a random walk with the same mean (7.6 percent) and standard deviation (19.5 percent) that they have displayed in the past.<sup>8</sup> As shown in Table 3, this calculation indicates that the market price for guaranteeing a floor equal to the risk-free real rate of return, 2 percent, amounts to 29 percent of a saver's annual contributions (see the Appendix for details on calculations).<sup>9</sup> In other words, for every dollar contributed to the savings plan, the participant would have to pay an additional 29 cents to the insurer.<sup>10</sup> Buying a floor at a 3-percent rate of return comes at a higher price, 46 percent of contributions. As the floor increases, the price of the insurance jumps sharply.

TABLE 3. PRICE OF GUARANTEED FLOORS AND CEILINGS WITH MARKET RISK AVERSION, PERCENT OF CONTRIBUTIONS

Rate of Return	Price of floor (1)	Price of ceiling (2)	Net cost for a collar (3)=(1)-(2)
2 percent	29%	29%	0%
3 percent	46	22	24
4 percent	71	16	56
5 percent	107	11	97
6 percent	157	7	150
7 percent	224	4	220

Source: Authors' calculations.

Savers can offset the cost of buying floors by surrendering some of their upside returns to the insurer. This ceiling for average returns is a kind of call option, and the participant can sell this option to the insurer. The insurer would then receive any returns that exceed the ceiling. Unlike the price of floors, the price of a ceiling falls rapidly as the ceiling increases because higher ceilings reduce the potential gain to the insurer (see Table 3, column 2). As a result, savers cannot expect to pay for generous floors by selling high ceilings.

The combination of a floor and a ceiling is known as a collar.<sup>11</sup> As shown in Table 3, the net price of a collar at the risk-free rate of return is zero – the cost of purchasing a floor is exactly covered by selling a ceiling.<sup>12</sup> Collars at higher rates of return involve much higher net costs. For example, a floor at 6 percent combined with a ceiling at 6 percent would cost 150 percent of the contribution – that is, \$1.50 for each \$1 dollar contributed. The net result – within this framework – is that the return on the *combined* insurance payment and contribution – the gross contribution – never exceeds the riskless rate.

If the participant is allowed some upside potential, say a floor of 2 percent and a ceiling of 3 percent, then the effective guarantee on the gross contribution must be less than the riskless rate (see Table 4). In short, an individual who wants any upside potential must pay for it.

The message from standard financial theory is that insurers cannot guarantee anything more than the riskless rate when they share the market's aversion to risk.<sup>13</sup> On the other hand, at least historically, a riskless rate guarantee would have done nothing to smooth out fluctuations since no cohort would have received a lifetime return of less than 3.2 percent.

This gap between past experience and finance theory's cap on feasible guarantees is due to limits on the market's willingness and ability to bear risk. This gap can be diminished if the government is less averse to risk than the market. Indeed, citizens acting in concert through the government can impose arrangements – such as intergenerational risk sharing – not possible by private agents acting on their own. In addition, credit-worthy governments can access capital

TABLE 4. PRICE OF COLLARS WITH MARKET RISK AVERSION

Floor/ Ceiling	Net price of collar	Gross contribution	Effective guarantee on gross contribution
2% / 2%	0%	\$1.00	2.00%
2% / 3%	7.0	1.07	1.68
3% / 3%	24.1	1.24	2.00
3% / 4%	30.3	1.30	1.77

Source: Authors' calculations.

markets at the risk-free rate of interest, provided their obligations are not excessive relative to their capacity to raise revenues now and in the future.<sup>14</sup> Experience also suggests that the government is more willing and able to bear macroeconomic risk than the private market. In its provision of unemployment insurance and its conduct of countercyclical fiscal policy, the government assumes commitments that appear too risky for private investors to provide. One question, then, is whether government guarantees should be priced according to the risk aversion evident in capital markets, or should they be priced with less aversion to risk?

Suppose, for example, the government's level of risk aversion were only one half that of private insurers. The price of a floor at the risk-free rate would be only 13 percent of contributions, less than half of its price using the market's aversion to risk (see Table 5).

TABLE 5. PRICE OF GUARANTEED FLOORS AND CEILINGS WITH HALF INDIVIDUALS' RISK AVERSION, PERCENT OF CONTRIBUTIONS

Rate of Return	Price of floor (1)	Price of ceiling (2)	Net cost for a collar (3)=(1)-(2)
2 percent	13%	97%	-84%
3 percent	23	83	-60
4 percent	40	68	-28
5 percent	66	53	13
6 percent	106	40	66
7 percent	165	28	137

Source: Authors' calculations.

More striking, by using collars, savers can purchase much higher guarantees at no, or little, net cost. For example, by purchasing a collar with a floor at a 4 percent rate of return and a ceiling at about 6 percent, savers would, at no net cost, be guaranteed an average rate of return on their retirement savings in this range.<sup>15</sup> The size of feasible guarantees, therefore, depends on the insurer's degree of risk aversion.<sup>16</sup>

## Conclusion

This *brief* has been a speculative discussion of what might be involved if a new tier of retirement saving were introduced and if large variations in replacement rates were viewed as unacceptable. The only way to avoid wide variations in replacement rates is to provide a guarantee or some form of risk sharing. And, when a guarantee is the method chosen, the only way to eliminate most of the variation is for the guarantee to be relatively high. Even though high guarantees would have been feasible historically, standard finance theory says that guaranteeing more than the riskless rate is impossible going forward. Finance theory also shows that the magnitude of feasible guarantees depends critically on insurers' capacity for bearing risk. When the government is less averse to risk than other investors, it can guarantee rates of return higher than the riskless rate. Consequently, the feasibility of providing attractive guarantees for returns in a new tier of savings accounts depends on whether applying private insurers' risk preferences to the government is appropriate.

## Endnotes

1 Munnell, Webb, and Golub-Sass (2008).

2 Munnell and Sundén (2006).

3 Munnell and Muldoon (2008).

4 Munnell, Webb, and Golub-Sass (2008).

5 A recent proposal would offer a guarantee with a real rate of return of 3 percent and allow a Board of Trustees to raise the guarantee if the economy performs better than expected (Ghilarducci 2008).

6 Although we are unaware of any governments that directly offer defined contribution pension plan guarantees, both Germany and Switzerland require fund managers to guarantee some level of investment returns (see Ammann (2003) and Maurer and Schlag (2002), respectively).

7 These balances assume that the individual began working in 1966 at a salary of \$19,200 in 2008 dollars, experienced annual wage growth of 2 percent, earning the reported average of \$44,000 for those employed at 65. The individual contributed 4 percent of salary each year, and received the return to U.S. equities each year.

8 This illustration could have used other distributions of returns, including distributions that specify some form of mean-reversion in returns or the price of equity. Replacing the random walk assumed here with mean-reverting returns would tend to compress the range of payoffs for pension balances, thereby reducing the cost of floors on average. However, if the reversion to mean is sufficiently slow, the potential cost of floors would be higher for participants who join the plan during years when returns are particularly low compared to the floor for those who join when returns are particularly high.

9 This analysis uses the simplifying assumption that the risk-free rate is a fixed real two percent. Lachance and Mitchell (2003) present a model in which the risk-free rate is a stochastic process.

10 Savers who purchase a floor at the risk-free rate can invest only \$1 per \$1.29 of contributions. Consequently, they are really guaranteeing only 78 percent

of the retirement wealth that they could attain by investing each dollar of contribution in risk-free bonds. Of course, their investment in equities offers an upside that risk-free bonds lack. Savers who purchase floors above the risk-free rate insure a higher rate of return, but their guaranteed minimum wealth in retirement falls as a result of the substantial cost of these floors.

11 See Feldstein and Rangelova (2001).

12 A collar at the risk-free rate fixes the return on stocks to match that on risk-free bonds. No-arbitrage features in standard financial theory require that investments with identical payoffs be priced the same. Consequently, an investment in stocks combined with this collar has a return and yield equivalent to an investment in risk-free assets.

13 The coefficient of relative risk aversion implied by the risk-free rate and distribution of returns on equity in this brief is 2. This figure rests at the low end of the range reported in the literature, which tends to cluster between 2 and 10 depending in part on whether the estimates are derived from portfolio theory, purchases of insurance, economic experiments, or preferences over lotteries (Chetty 2003). Three factors are important in considering the reasonableness of the implied level of risk aversion in this *brief*. First, the 2 applies only to the marginal investor; infra-marginal investors could well have higher levels of risk aversion. Second, the market pricing of securities reflects the risk aversion of active institutional and professional investors, who are likely to be more willing and able to bear risk than the average investor. And, third, the pricing of securities likely reflects the assessments of investors who are most optimistic about future returns, which tends to depress the coefficient of risk aversion implied by market pricing.

14 Gollier (2008) shows that the ability of governments to enforce intergenerational risk sharing can increase the certainty equivalent rate of return by allowing pension funds to invest their financial assets more aggressively than would otherwise be optimal.

15 When an insurer's aversion to risk is less than that in the market, the insurer can capture a share of the market's risk premium in the return on equity. When the insurer is not averse to risk, it can offer the entire premium, at no extra cost, in guarantees of returns.



As a result, whereas market pricing does not allow guarantees in excess of the risk-free rate, an insurer with no aversion to risk could offer guarantees approaching 7.6 percent at no net cost.

<sup>16</sup> A stable and responsible central government is likely less averse to risk than its households and businesses. This *brief* finds that the coefficient of relative risk aversion is about 2 for marginal private investors, given its assumptions about the risk-free rate and the distribution of the returns on equity. Older public finance literature defended a low social discount rate on public investment, reflecting the government's broad scope for diversifying risk and its risk-free cost of capital (Samuelson 1964; Vickery 1964; and Arrow 1965, 1966), thereby implying a low coefficient of risk aversion in its pricing of investments from the viewpoint of the private market. Some more recent work suggests discount rates for public investments that could exceed those of private investments (Jensen and Bailey 1972; and Bazelon and Smetters 1999). But, when the public investment covers the risk in the entire market portfolio averaged over long horizons, and the government can access capital, even in difficult times, at the risk-free rate, then the case for an effectively low coefficient of risk aversion for the government is more compelling. This *brief* uses a value of 1 simply to illustrate the potential magnitude of the public benefit that the government might provide.

<sup>17</sup> See Black and Scholes (1973).

<sup>18</sup> See LaChance and Mitchell (2003).

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# APPENDIX

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## Retrospective Guarantees – Tables 1 and 2

To compute when payments are required, we compare two types of accounts at retirement: a guaranteed account and a non-guaranteed account. In each case, workers are assumed to enter the labor force at age 22, have earnings rise 2 percent a year, contribute 4 percent of earnings into the account, and retire after 43 years, at 65. Each year, the guaranteed account earns the level of guarantee specified in the model. The non-guaranteed account earns the return experienced by U.S. equities each year for the individual's 43-year worklife.

To calculate the values in Table 1, we determine whether the non-guaranteed account's final balance falls short of the guaranteed account's final balance. To calculate the amount of the payment, we need to determine the final wage of each cohort. For the years after 1962, we use the *Current Population Survey* (CPS) to calculate the average wage for individuals at age 56 for that particular cohort. (We use this procedure because a substantial segment of the age 65 population does not receive a wage.) We then project forward to age 65 using the model's assumed annual growth rate of 2 percent. Before 1962, we assume that the fraction of total wages earned by individuals aged 56 before 1962 equals the average fraction of total wages earned by individuals aged 56 after 1962. For each year before 1962, we divide this fraction of total wages by the total population of individuals age 56 to find the average wage. Again, because the CPS began in 1962, we must estimate the age 65 population for all preceding years. Age 65 populations are calculated using the weighted averages of forecasts and backcasts made from the Decennial Census reports issued before and after the year in question, respectively, and Social Security Administration cohort life tables. (Note: all values are presented in chained year 2000 dollars). Finally, this value is then divided by Gross Domestic Product (GDP) for each year in which a payment needed to be made on a guarantee. Table 2 is based on the values calculated for Table 1.

For column 1, we calculate the net gain or loss in each year as a percent of GDP. We then take the average of those gains and losses over the entire 84-year period. For columns 2 and 3, we find the sum of the net losses as a percent of wages and GDP, respectively, and divide by 84 – the total number of years.

## Prospective Guarantees – Tables 3, 4, and 5

The guarantee of a minimum return for a participant in a defined contribution pension plan is a kind of put option. We value this option with Monte Carlo simulations instead of the Black-Scholes equation because the option's payoff depends on the path taken by returns.<sup>17</sup> The sequence of returns is critical when the option covers the average rate of return earned on a portfolio that receives annual contributions.<sup>18</sup>

For each of the 10,000 simulations, an individual makes a yearly contribution equal to a constant percent of his wages, the individual's contribution is added to his assets and the balance earns a return. The terminal balance  $b_T$ , will be equal to

$$b_T = \sum_{t=1}^{43} c_t \prod_{j=t}^{43} (1+r_j); c_t = c_1 (1+w)^{t-1} \quad (1)$$

where  $c_1$  is the initial contribution,  $w$  is the assumed rate of wage growth, and  $r_t$  is the return experienced in year  $t$ . Returns were randomly generated as independent and identically distributed normal variables with a mean of 7.6 percent and a standard deviation of 19.5 percent.

Each of the 10,000 terminal balances is sorted, arrayed, and assigned a probability of 1 in 10,000 to form the actual probability density function (PDF). The analysis examines guarantors whose aversion to risk matches that of the market (Table 3) and those who are less averse to risk (Table 5).

Assuming guarantors are averse to risk, we value the payoffs according to a constant relative risk aversion (CRRA) utility function with a coefficient of risk aversion of 2.02. The coefficient of risk aversion is consistent with investors' requiring a 5.6 percent risk premium to hold a risky asset for 43 years with the normally distributed returns described above. The risk-neutral PDF for these guarantors is the product of the actual PDF for terminal balances derived from the Monte Carlo simulations and the pricing kernel (the marginal utility of wealth divided by consumption), which is then normalized to sum to one.

The entries in Table 5 assume that guarantors are less averse to risk. The risk-neutral PDF in this case is the product of the actual PDF and the pricing kernel derived from a CRRA utility function with a coefficient of risk aversion of 1.0.

To calculate the cost of a floor, we first use equation (1) to calculate the risk-free terminal wealth that would result from receiving the risk-free return each year. We then use equation (1) to calculate the guaranteed terminal wealth that would result from receiving the guaranteed rate of return each year. The cost of the floor is the present value of the expected value of terminal balances that are less than guaranteed terminal wealth. The cost of the floor per dollar of contributions equals the conditional expectation of terminal balances that are less than guaranteed wealth divided by risk-free terminal wealth. To calculate this conditional expectation, the entries in Table 3 use the risk-neutral PDF that incorporates a coefficient of risk aversion of 2.02, while those in Table 5 use the risk-neutral PDF that incorporates a coefficient of risk aversion of 1.

To calculate the cost of a ceiling, we use equation (1) to calculate the maximal terminal wealth that would result from receiving the ceiling rate of return each year. The cost of the ceiling is the present value of the expected terminal balances that exceed maximal terminal wealth. The cost of the ceiling per dollar of contributions equals the conditional expectation of terminal balances that exceed the maximal balance divided by risk-free terminal wealth. Again, the entries in Table 3 use the risk-neutral PDF that incorporates a coefficient of risk aversion of 2.02, while those in Table 5 use the risk-neutral PDF that incorporates a coefficient of risk aversion of 1.0.

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The Center for Retirement Research at Boston College was established in 1998 through a grant from the Social Security Administration. The Center's mission is to produce first-class research and forge a strong link between the academic community and decision makers in the public and private sectors around an issue of critical importance to the nation's future. To achieve this mission, the Center sponsors a wide variety of research projects, transmits new findings to a broad audience, trains new scholars, and broadens access to valuable data sources. Since its inception, the Center has established a reputation as an authoritative source of information on all major aspects of the retirement income debate.

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